Calorimetry and Heat Transfer

ELEMENTARY

Q.1 (1)

Water equivalent = m × c = 400 × 0.1 = 40g

Q.2 (2)

Resultant temperature is 0°C while ice will not melt.

Q.3 (4)

Utensil should have low thermal resistan ce $\left(R = \dfrac{\ell}{KA}\right)$

and low specific heat so that heat loss is less

Q.4 (3)

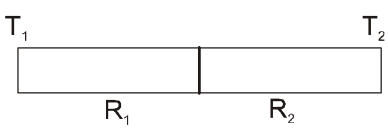
$$\frac{R_1}{R_2} = \frac{\dfrac{\ell_1}{K_1A_1}}{\dfrac{\ell_2}{K_2A_2}} = \frac{\dfrac{\ell}{K\pi(2r)^2}}{\dfrac{2\ell}{K\pi(3r)^2}} = \frac{9}{8}$$

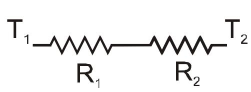
$$\because\ I = \frac{\Delta T}{R} \Rightarrow I \propto \frac{1}{R}$$

$$\text{so } \frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{8}{9}$$

Q.5 (2)

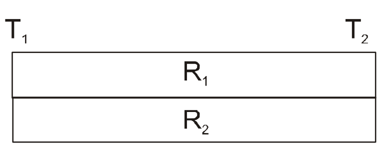
Q.6 (2)

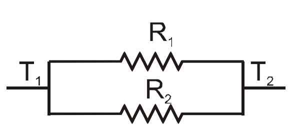


Equivalent thermal circuit 

$$R_{eq} = R_1 + R_2 = \frac{2\ell}{KA} = \frac{\ell}{K_1A} + \frac{\ell}{K_2A} \Rightarrow K = \frac{2K_1K_2}{K_1 + K_2}$$

Q.7 (3)



Equivalent thermal circuit 

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{K_{eq} \times 2A}{\ell} = \frac{KA}{\ell} + \frac{2KA}{\ell}$$

$$\Rightarrow K_{eq} = \frac{3}{2}K$$

Q.8 (4)

Given $A_1 = A_2$ and $\dfrac{K_1}{K_2} = \dfrac{5}{4}$

$$\because R_1 = R_2 \Rightarrow \frac{l_1}{K_1A} = \frac{l_2}{K_2A} \Rightarrow \frac{l_1}{l_2} = \frac{K_1}{K_2} = \frac{5}{4}$$

Q.9 (2)

Because of uneven surfaces of mountains, most of it's parts remain under shadow. So, most of the mountains. Land is not heated up by sun rays. Besides this, sun rays fall slanting on the mountains and are spread over a larger area. So, the heat received by the mountains top per unit area is less and they are less heated compared to planes (Foot).

Q.10 (4)

According to Kirchoff's law in spectroscopy. If a substance emit certain wavelengths at high temperature, it absorbs the same wavelength at comparatively lower temperature.

Q.11 (2)

$$\lambda_{m_2} = \frac{T_1}{T_2} \times \lambda_{m_1} = \frac{2000}{3000} \times \lambda_{m_1} = \frac{2}{3}\lambda_{m_1} = \frac{2}{3}\lambda_m$$

Q.12 (1)

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow \frac{E}{E_2} = \left(\frac{273+0}{273+273}\right)^4 \Rightarrow E_2 = 16\,E$$

Q13 (1)

$$E \propto T^4 \Rightarrow \frac{E_1}{E_2} = \frac{T^4}{T^4} \times 2^4 \Rightarrow E_2 = \frac{E}{16}$$

Q.14 (1)

From Stefan's law $E = \sigma T^4$

$$T^4 = \frac{E}{\sigma} = \frac{6.3 \times 10^7}{5.7 \times 10^8} = 1.105 \times 10^{15} = 0.1105 \times 10^{16}$$

$T = 0.58 \times 10^4\ K = 5.8 \times 10^3\ K$

Q.15 (2)

According to Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

In the first case,

$$\frac{(60-50)}{10} = K\left[\frac{60+50}{2} - \theta_0\right]$$

$1 = K(55 - \theta)$ ….(i)

In the second case,

$$\frac{(50-42)}{10} = K\left[\frac{50+42}{2} - \theta_0\right]$$

$0.8 = K(46 - \theta_0)$ ….(ii)

Dividing (i) by (ii), we get

$$\frac{1}{0.8} = \frac{55 - \theta_0}{46 - \theta_0}$$

or $46 - \theta_0 = 44 - 0.8\,\theta_0 \Rightarrow \theta_0 = 10°C$

Q.16 (1)

For small difference of temperature, it is the special case of Stefan's law.

Q.17 (3)

In first case

$$\frac{60-40}{7} = K\left[\frac{60+40}{2} - 10\right] \quad .....(i)$$

In second case

$$\frac{40-28}{t} = K\left[\frac{40+28}{2} - 10\right] \quad ….(ii)$$

By solving t = 7 minutes

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

$$mc\theta = m_i L \Rightarrow m_i = \frac{mc\theta}{L}$$

Q.2 (4)

Heat is required to raise temperature of

(Calorimeter + Ice to vapour) $_{\text{at 0 to 100°C}}$

$= (10\times100 + \{10\times80 + 10\times1\times100 + 10\times540\})$

$= 8200$ Cal.

Q.3 (1)

Required heat/sec = 0.1×80 cal/gm = 8 cal/sec

Produced mass = $0.1\times100 = 10$ gm ice or water [now $Q = ms\Delta T$]

In unit time rise of temperature will be

$\Delta T = Q/ms = 8/(10\times1) = 0.8°C/s$

$R = 0.1 \times 80 = 8$ cal/sec.

Q.4 (1)

Using Energy conservation

The energy loss due to potential energy goes into increasing the temperature of ice.

$$\frac{m}{5}(L) = mgh$$

$$\Rightarrow \quad h = \frac{L}{5g}$$

Q.5 (4)

From the data given

$S_A\rho_A(8V) = (12V)\,\rho_B s_B$

$$\frac{s_A}{s_B} = \frac{12\rho_B}{8\rho_A} = \frac{3}{2}\times\frac{2000}{1500} = 2$$

Q.6 (1)

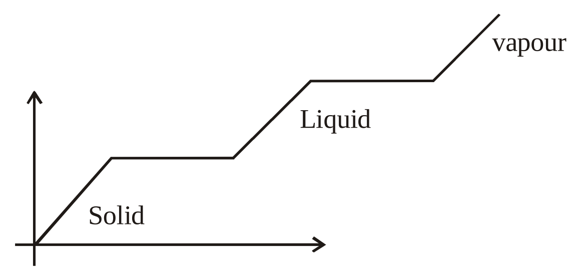
Let m is the mass

$mL_v + ms_w(100-80) = (1.1 + 0.02)s_w(80-15)$

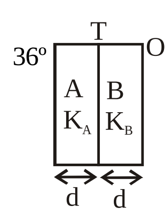
$m(540+20) = (1.12)\,65 \Rightarrow m = 0.130$ kg

Q.7 (4)

$$\therefore\ dQ = msdT \Rightarrow \frac{dT}{dQ} = \frac{1}{ms}$$



Q.8 (2)



$K_A = 2K_B = 2\,K$

$\left(\dfrac{36-T}{d}\right) K_A A = \left(\dfrac{T-0}{d}\right) K_B A$

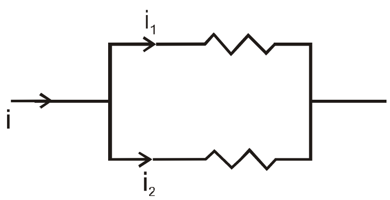
$(36-T)\,2\,K = T\,K$

$T = \dfrac{72}{3} = 24$

$\Delta T = \text{temp diff} = 36 - 24 = 12$

Q.9 (a)(1), (b)(4)

$i_1 = \dfrac{(100-20)}{3\times10^{-2}}(209)\,9\times10^{-4}$



$i_2 = \dfrac{100-20}{3\times10^{-2}}\,(385)\,9\times10^{-4}$

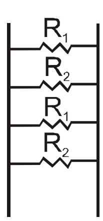
$i_T = i_1 + i_2 = 1.42\times10^3\ w$

$\dfrac{i_{Cu}}{i_{Al}} = \dfrac{i_2}{i_1} = \dfrac{385}{209}$

Q.10 (2)

It`s a parallel Combination

$R_1 = \dfrac{d}{K_1 A}$ $\qquad$ $R_2 = \dfrac{d}{K_2 A}$



$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots\ldots\ldots\ldots\ldots\ldots$upto n^{th}

$\dfrac{1}{R_{eq}} = \dfrac{n}{2R_1} + \dfrac{n}{2R_2} = \dfrac{n}{2}\left(\dfrac{R_1+R_2}{R_1R_2}\right)$

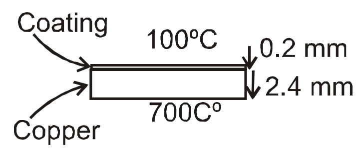
$R_{eq} = \dfrac{2(R_1R_2)}{n(R_1+R_2)} \Rightarrow \dfrac{d}{K_{eq}(nA)} = \dfrac{2\left(\dfrac{d}{K_1A}\right)\times\dfrac{d}{K_2A}}{n\dfrac{d}{A}\left(\dfrac{1}{K_1}+\dfrac{1}{K_2}\right)}$

$\Rightarrow K_{eq} = \dfrac{K_1+K_2}{2}$

Q.11 (3)

$i_H = \dfrac{\Delta T}{R_{eq}} = \dfrac{700-100}{R_1+R_2}$

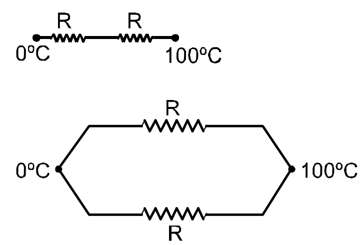
Where $R_{eq} = R_1 + R_2 = \dfrac{0.24}{0.9\times400} + \dfrac{0.02}{0.15\times400}$



$i_H = \dfrac{dQ}{dt} = \dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta m.L}{\Delta t}$

$\dfrac{\Delta m}{\Delta t} = \dfrac{i_H}{L}$ where $L = 540$ cal/gm ; $\Delta t = 3600$ sec.

Q.12 (2)



$\dfrac{Q_1}{t_1} = i_{H_1} = \dfrac{100-0}{2R} = \dfrac{50}{R}$

$i_{H_2} = \dfrac{100}{R/2} = \dfrac{200}{R} = \dfrac{Q_2}{t_2}$

$Q_1 = Q_2 = 10$ cal.

$\dfrac{50}{R}\times(2) = \dfrac{200}{R}\times t_2$

$t_2 = \dfrac{1}{2}$ min.

Q.13 (1)

The heat current in the bottom of pot is due to temperature difference at the lower & upper surface.

$i_p = K_{steel}A.\dfrac{dT}{dx} = \dfrac{m}{t}.\ L_v$

$50.2\times0.15\times\dfrac{(x-100)}{1.2\times10^{-2}} = \dfrac{0.44}{5\times60}\times2.25\times10^6$

[Let x be temperature of surface in contact with stove]

$x = 105.25°C$

Q.14 (3)

The heat current is equal to the heat required for fusion of ice per dt time.

$$i = \frac{dm}{dt}.L_f = KA\left(\frac{20-0}{2.35}\right)$$

$$\frac{dm}{dt} = 2.4\pi \times 10^{-6}$$

Q.15 (1)

We know that

$$i = K(\pi R^2)\frac{dT}{dx}, i \propto \frac{R^2}{\ell}$$

Q.16 (2)

We know that

i = – kAdT / dx

And slope of the curve but dT/dx = – i/kA

i is constant (steady state), A is constant but since k is decreasing from 2k to k, hence slope is –ve but less –ve to more –ve.

Q.17 (1)

From the given condition as the plates are in series so heat current is same.

$$i_1 = i_2 \Rightarrow k_1A\frac{T_B - T_A}{d} = \frac{k_2A(T_c - T_B)}{2d}$$

$$\frac{k_1}{k_2} = \frac{T_C - T_B}{2(T_B - T_A)} = \frac{1}{2}\left(\frac{4T_A - 2T_A}{2T_A - T_A}\right) = 1$$

Q.18 (4)

$$i = kA\frac{dT}{dx} \Rightarrow \frac{dT}{dx} \propto \frac{1}{K}$$

∴ i and A are same for both the layers.

i = – kA (dT/dx)

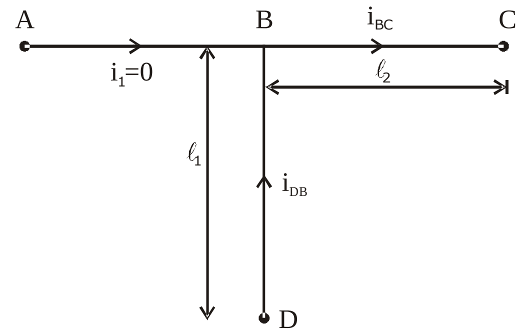
i and A are constant hence slope

dT/dx = –i/(kA) is – ve but

Slope ∝ (1/k)

Hence in air slope will be more – ve due to very less conductivity.

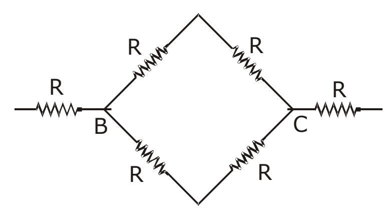
Q.19 (2)



$$i_{BC} = i_{DB} \Rightarrow \frac{kA(90-20)}{\ell_1} = \frac{kA(20-0)}{\ell_2}$$

$$\frac{\ell_1}{\ell_2} = \frac{7}{2}$$

Q.20 (3)



$$T_C - 20 = T_B - T_C = T_A - T_B = \frac{200-20}{3} = 60$$

$T_C = 80$

So $T_B = 80 + 60 = 140$ °C

Q.21 (2)

The heat current is equal to required latent heat of fusion per unit time.

$$i = \frac{dm_{ice}}{dt}.L_f = \frac{kA(100)}{\ell}$$

$$k = \frac{dm_{ice}}{dt}.\frac{\ell L_f}{A(100)} = 60\ Wm^{-1}k^{-1}$$

Q.22 (3)

i = –kA dT / dx

Slope dT/dx = – i/kA is – ve but due to radiation loss because of not lagged, as we move ahead current i will be less. Hence slope wil be more – ve to less – ve.

Q.23 (1)

$$T_p = \frac{100+0}{2} = 50°$$

As $T_P > T_Q$ so flow is from P to Q.

$$T_Q = \frac{30+60}{2} = 45°$$

Q.24 (1)

Slope dT/dX = – i/kA is less – ve for 1st layer Hence 1st layer should have larger k.

So $k_1 > k_2$

Q.25 (1)

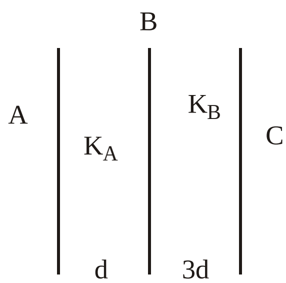
Consider the two sections like two resistance R_1 & R_2.

$$R_A = \frac{\ell_1}{k_1 A}\; R_B = \frac{2\ell_1}{\frac{k_1}{2}A}$$

$$\text{So}\quad \theta = \left[\frac{R_B}{R_A + R_B}\right][100-0]$$

$\theta = 80°\,C$

Q.26 (1)



Q.27 (3)

$$\text{Initially } i = \frac{dm}{dt}\,.\,L_f = k\pi R^2.\,\frac{100}{\ell}$$

$$\text{Hence } \frac{dm}{dt} \propto \frac{kR^2}{\ell}$$

From given condition

$$\frac{\frac{dm_2}{dt}}{\frac{dm_1}{dt}} = \frac{\frac{k}{4}\left(\frac{(2R)^2}{\ell/2}\right)}{\frac{kR^2}{\ell}}$$

$$\frac{\frac{dm_2}{dt}}{0.1} = 2 \Rightarrow \frac{dm_2}{dt} = 0.2$$

Q.28 (1)

As the heat current through all the rods is same. So all the resistance are in series.

$R_{eq} = R_1 + R_2 + R_3$

$$\frac{3\ell}{k_{eq}A} = \frac{\ell}{\frac{k}{2}A} + \frac{\ell}{5kA} + \frac{\ell}{kA}$$

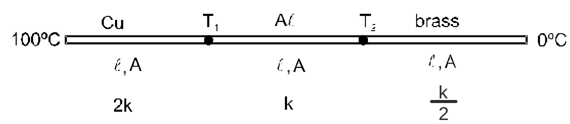
$$\frac{3}{k_{eq}A} = \frac{2}{k} + \frac{1}{5k} + \frac{1}{k} = \frac{16}{5k}$$

$$k_{eq} = \frac{15}{16}k$$

Q.29 (1)

Req. is same for both the rods and same temperature same difference so $i_1 = i_2$

Q.30 **(4)**



$R_{eq} = R_1 + R_2 + R_3$ where $R_1 = \frac{\ell}{(2k)A}$, $R_2 = \frac{\ell}{kA}$, R_3

$$\frac{\ell}{\left(\frac{k}{2}\right)A}$$

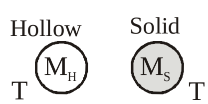
$$\frac{100-0}{R_{eq}} = \frac{100-T_1}{R_1} = \frac{100-T_2}{R_1+R_2} = \frac{T_2-0}{R_3}$$

Q.31 (1)

$P_{emitte} = \sigma\, eAT^4$

since $T_1 = T_2$

$P_{absorb} = \sigma\, eAT_S^4$



$M_H < M_S$

So, $P_1 = P_2$ at $t = 0$

cooling rate $\left(-\frac{dT}{dt}\right) = \frac{\sigma eA}{mS}[T^4 - T_S^4]$

since $M_H < M_S$, so cooling rate will be different since cooling rate is not same so both will not have same temp at any instant t (except t = 0)

Q.32 (2)

$$\left(-\frac{dT}{dt}\right) = \frac{\sigma eA}{mS}[T^4 - T_S^4]$$

Rate of temperature fall will be maximum when ($T^4 - T_S^4$) has mass value i.e. T has max. value

$\left(-\frac{dT}{df}\right)_{max} = \frac{\sigma eA}{mS}[500^4 - 300^4]$ Put all values & get answer.

Q.33 (3)

For small temperature difference, Stefan's law can written as

$$\Delta u = e\sigma A[(T + \Delta T)^4 - T^4]$$

or $\Delta u = e\sigma AT^4 \left[\left[1 + \frac{\Delta T}{T}\right]^4 - 1\right]$

or $\Delta u = e\sigma AT^4 \times 4 \times \frac{\Delta T}{T}$

or $\Delta u \propto \Delta T$

Hence Newton's law of cooling is a special case of stefan's law.

Q.34 (4)

Power

$$P = \frac{dQ}{dt} = A\sigma T^4 = A\sigma\left(\frac{b}{\lambda}\right)^4$$

$$\frac{P_2}{P_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^4 = \left(\frac{\lambda_0}{3/4\,\lambda_0}\right)^4 = \frac{256}{81}$$

Q.35 (2)

Let $I = \frac{P'}{4\pi d^2}$ or $I = \frac{e\sigma AT^4}{4\pi d^2}$

and $I A_f = P$ (Given)

Now $P_{new} = I_{new} A_f = \frac{e\sigma A(2T)^4}{4\pi(2d)^2}.A_f$

$$= \frac{16}{4}\left[\frac{e\sigma AT^4}{4\pi d^2}.A_f\right] = \frac{16}{4}P$$

Q.36 (1)

We know that

$$\lambda_{max} \propto \frac{1}{T}$$

$$\frac{\lambda_{1\,max}}{\lambda_{2\,max}} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{3}{4}$$

Q.37 (3)

$$-\frac{dT_p}{dt} = x\left(-\frac{dT_Q}{dt}\right)$$

$$\Rightarrow \frac{eA_p\sigma(T^4 - T_0^4)}{m_pS} = \frac{xe\sigma A_Q(T^4 - T_0^4)}{m_QS}$$

$$\Rightarrow x = \frac{A_p m_Q}{A_Q m_p} = \left(\frac{r}{3r}\right)^2 \times \left(\frac{3r}{r}\right)^3$$

$$\Rightarrow x = 3$$

Q.38 (2)

Initially the temperature of the substance increases and then phase change from ice to water occurs & this process continues.

Q.39 (4)

$$\text{Area} = \int y\,dx = \int \frac{dE}{d\lambda} \times d\lambda = \int dE$$

$$\text{Area (1)} = E = \sigma T^4 = \sigma\left(\frac{b}{\lambda}\right)^4$$

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4 \Rightarrow \frac{1}{9} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{3}$$

Q.40 (2)

Using relation $\lambda_{max} \propto \frac{1}{T}$

$$\frac{T_S}{T_{NS}} = \frac{\lambda_{NS_{max}}}{\lambda_{S_{max}}} = \frac{350}{510} = 0.69$$

Q.41 (2)

Using formula

$P = \sigma eAT^4$

$P_P = \varepsilon_P \sigma (1) \theta_P^4$ and $P_Q = \varepsilon_Q \sigma A \theta_Q^4$

Now $P_P = P_Q$

$$\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4} \theta_Q = \theta_P$$

Q.42 (1)

$E_{273} = eA(273+273)^4$

$= E$(Given)

$E_0 = eA(273+0)^4$

$$E_0 = \frac{E}{16}$$

Q.43 (2)

$\because$ Rate of cooling, $y = (T - T_0)\,k$ (from Newton's law of cooling)

T_0 : surrounding temperature

k : +ve constant

$\Rightarrow$ graph is straight line with +ve slope

Q.44 (4)

$$i = ms\frac{d\theta}{dt} = msk(50° – 20°) = 10\text{ W} \quad ..(1)$$

and $\frac{35.1-34.9}{60} = k(35-20)$ ...(2)

from (1) & (2)

$$\frac{0.2}{60} = \frac{10}{ms(30)} \times 15$$

ms = 1500 J/°C

Q.45 (2)

If the body cools from θ_1 to θ_2 then using formula

$$\frac{\theta_1 - \theta_2}{t} = \alpha\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\frac{60-50}{4} = k\left(\frac{60+50}{2} - \theta_0\right)$$

$$\frac{5}{2} = k(55 - \theta_0) \quad ...(1)$$

and $\frac{40-30}{8} = k\left(\frac{40+30}{2} - \theta_0\right)$

$$\frac{5}{4} = k(35 - \theta_0) \quad ..(2)$$

from (1) & (2)

$$2 = \frac{55-\theta_0}{35-\theta_0}$$

$\theta_0 = 70 - 55 = 15°C$

Q.46 (1)

If the body cools from θ_1 to θ_2 then using formula

$$\frac{\theta_1 - \theta_2}{t} = \alpha\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\frac{75-65}{5} = k\left(\frac{75+65}{2} - 25\right)$$

$$2 = K(70-25) \Rightarrow K = \frac{2}{45}$$

Now $\frac{65-x}{5} = k\left(\frac{65+x}{2} - 25\right)$

$2(65 - x) = 5k(65 + x - 50)$

$$130 - 2x = 5 \times \frac{2}{45}(15 + x)$$

x = 57°c

Q.47 (3)

$$\frac{40-36}{5} = k\left(\frac{40+36}{2} - 16\right)$$

$$\frac{4}{5} = K(38 - 16)$$

$$\Rightarrow k = \frac{2}{55}$$

..(1)

$$\frac{36-32}{t} = \frac{2}{55}\left(\frac{36+32}{2} - 16\right)$$

$$\frac{2 \times 55}{t} = (34 - 16)$$

t = 6.1 min

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

Water flow rate = 20 gm/sec

for 1 sec

$Q = P \times t = 2100 \times 1 = 2100$ J

$Q = 2100 = 20 \times 4.2\,(t-10)$

$t = 35°C$

Q.2 (B)

For 1 sec we can say that

$P_c \times 80\,\% = (\rho v)\, s\,(t-10)$

$$2\times10^3 \times \frac{80}{100} = (1000).100\times(10^{-2})^3.4200\,(t\text{-}10).$$

On solving

$t = 13.8\ °C$

Q.3 (A)

$(m_w + w_f)\,(1)\,(70\text{–}40) = m_{ice}L_f + m_{ice}(1)(40\text{-}0)$

$(200+w_f)\,(70\text{–}40) = 500\,L_f + 50\times40$ ..(1)

$(m_w + w_f + m_{ice})\,(40\text{–}10) = m_{ice}L_f + m'_{ice}\,(1)(10\text{–}0)$

$(200+w+50)\,30 = 80\,L_f + 80\times10$ ..(2)

from eq. (1) & (2)

$50\times30 = 30\,L_f - 30\times40$

$L_f = 90$ cal/gm $= 3.78\times10^5$ J/kg

Q.4 (B)

At a temperature T

$dQ = SdT = aT^3dT$

$$Q = a\int_1^2 T^3 dT = \frac{a[T^4]_1^2}{4} = \frac{15a}{4}$$

Q.5 (C)

For vapourization the total time required is

= (30 – 20) min = 10 min

Total Heat Given = 42 KtJ × 10 = 420 KJ

so mL = 420 kJ

$5L = 420 \Rightarrow L = 84$ KJ/kg

Q.6 (C)

Ice Changes to water hence volume decreases but mass remains same hence

$V_w P_w = V_{ice} P_{ice}$

$$V_w = \frac{V_{ice}P_{ice}}{P_w}$$

Let volume (V_{ice}) change to water

$(0.9\,\rho_w V_{ice})L = H$

...(1)

$$\Delta v = v_{ice} - v_w = \left(v_{ice} - \frac{v_{ice}\,\rho_{ice}}{\rho_w}\right)$$

$= v_{ice}\,(1-0.9) = 0.1\,v_{ice} = 1\ cm^3$

$v_{ice} = 10\ cm^3$

So from eq. (1)

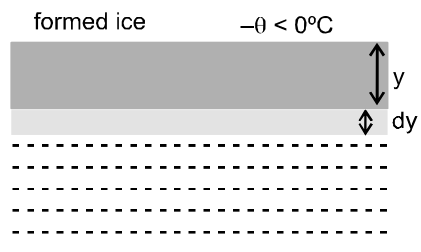
$[0.9\times1\times10]\times80 = H$

H = 720 cal.

Q.7 (C)

$$i_H = \frac{0-(-\theta)}{(y/KA)} = \frac{\theta\, KA}{y} = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} = L\frac{dm}{dt} = L\frac{\rho.A\,dy}{dt}$$



$$\frac{KA\theta}{y} = \rho\, AL\frac{dy}{dt}$$

$$\int_2^4 y\,dy = \int_0^{3600}\left(\frac{K\theta}{\rho L}\right)dt$$

$$\left(\frac{y^2}{2}\right)_2^4 = \frac{K\theta}{\rho L}\,(t)_0^{3600}$$

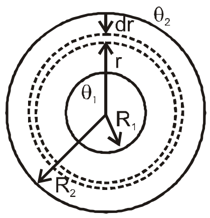
Where $K = 4\times10^{-3}$

$\rho = 0.9$ gm/cc

$L = 80$ cal/gm

Q.8 (C)

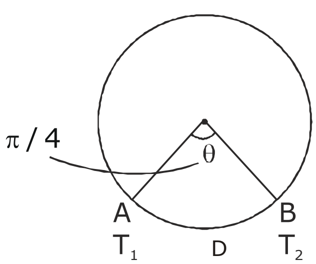
$$\theta_1 - \theta_2 = \Delta\theta\ \frac{\theta_1 - \theta}{\int_{R_1}^{R}\frac{dr}{K4\pi r^2}} = \frac{\theta_1 - \theta_2}{\int_{R_1}^{R_2}\frac{dr}{K\,4\pi r^2}}$$



$$\frac{\Delta\theta/2}{\frac{1}{4\pi K_1}\left[\frac{1}{R_1}-\frac{1}{R}\right]} = \frac{\Delta\theta}{\frac{1}{4\pi K_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

$$\Rightarrow R = \frac{2R_1R_2}{R_1+R_2}$$

Q.9 (A)



Initially

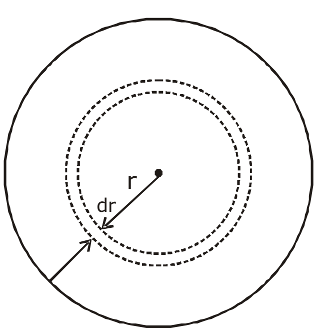
$$H = \frac{kA(T_2-T_1)}{(2\pi-\theta)R} + \frac{kA(T_2-T_1)}{\theta R} \quad ...(1)$$

finally $2H = \frac{kA(T_2-T_1)}{(2\pi-\theta)R} + \frac{k'A(T_2-T_1)}{\theta R} \quad ...(2)$

from (1) & (2) $k' = \frac{4k}{3} + k = \frac{7k}{3}$

Q.10 (C)

$$\int dR = \int_{r_1}^{r_2} \frac{dr}{k(4\pi r^2)}$$

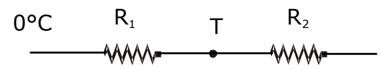


$$R_{eq} = \frac{1}{4\pi k}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Now R_1 (when $r_1 = R, r_2 = 2R$) $= \frac{1}{8\pi kR}$

and R_2 (when $r_1 = 2R, r_2 = 3R$) $= \frac{1}{4\pi kR}\left[\frac{1}{2}-\frac{1}{3}\right]$

$$= \frac{1}{24\pi kR}$$



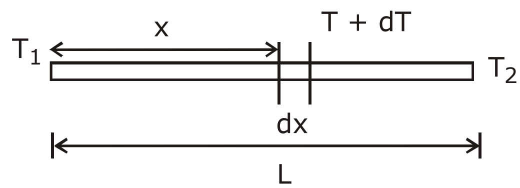
$$T = \frac{R_1}{R_1+R_2} \times 100 = 75°\,C$$

Q.11 (A)

Taking an element at a distance x of length dx and having at temperature difference dT.

$$i = \frac{\alpha}{T} A \frac{dT}{dx} = C \text{ (const.)}$$

$$\Rightarrow \alpha A[\ell n T]_{T_1}^{T} = Cx$$

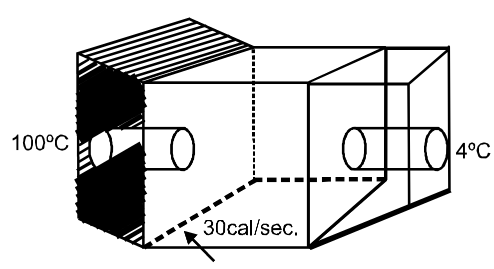


$$\alpha \ell n\left(\frac{T}{T_1}\right) = \left(\frac{C}{A}\right)x$$

at $x = L, T = T_2 \Rightarrow \frac{C}{A} = \frac{\alpha}{L} \ln \frac{T_2}{T_1}$

So $T = T_1\left(\frac{T_2}{T_1}\right)^{x/L}$

Q.12 (C)

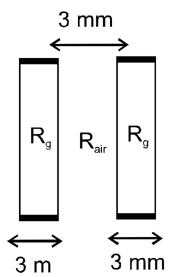


$$36 = \left(\frac{T-100}{8}\right)kA + \left(\frac{T-4}{8}\right)kA$$

K = 0.5 cal/°c/cm

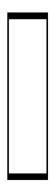
A = 12 cm^2.

Q.13 (D)



(A)

$$R_{eq} = 2\,R_g + R_{air} = \frac{2(3mm)}{k_g A} + \frac{(3mm)}{k_{air}\,A}$$



$$R = \frac{6mm}{k_g.A}$$

(B)

$$\frac{i_A}{i_B} = \frac{\frac{\Delta T}{R_{eq}}}{\frac{\Delta T}{R}} = \frac{R}{R_{eq}} = \frac{\frac{6mm}{K_g.A}}{\frac{2(3mm)}{K_g\ A}+\left(\frac{3mm}{K_{air}\ A}\right)} = \frac{\frac{1}{K_g}}{\frac{2K_{air}+K_g}{2K_g.K_{air}}}$$

$$= \frac{2K_{air}}{K_g + 2K_{air}}$$

Q.14 (B)

Let at time t radius be r

Then $\dfrac{dQ}{dt} = CA = 4C\pi r^2 = -\dfrac{dm}{dt}.L_f$

$m = \rho_{ice}\,\dfrac{4}{3}\,\pi r^3 \Rightarrow dm = C_0 r^2\,dr$

So. $(4\,C\pi)\,r^2 = -L_f\,C_0\,r^2\,\dfrac{dr}{dt} \quad \Rightarrow \dfrac{dr}{dt} = \text{const}$

Q.15 (B)

$H = \sigma\,e\,A\,T^4$

$H\,\alpha\,A\ \alpha\,r^2$

$C = \dfrac{\sigma e A}{ms}\ (4T_S^{\,3}\,\Delta T) \qquad C\,\alpha\,\dfrac{A}{m}\ \alpha\,\dfrac{r^2}{r^3}\ \alpha\,r$

Q.16 (D)

$e_A : e_B : e_C = 1 : \dfrac{1}{2} : \dfrac{1}{4}$

Rate of emission : $\dfrac{dQ}{dt} = eA\sigma T^4$ is same

So, eT^4 is same $\quad \Rightarrow T_A^4 : T_B^4 : T_C^4 = \dfrac{1}{e_A} : \dfrac{1}{e_B} : \dfrac{1}{e_C} = 1:2:4$

as $\lambda T = b$ = constant

So, $\lambda_A^4 : \lambda_B^4 : \lambda_C^4 = \dfrac{1}{T_A^4} : \dfrac{1}{T_B^4} : \dfrac{1}{T_C^4}$

$= 1 : \dfrac{1}{2} : \dfrac{1}{4}$

On solving $\sqrt{e_A\lambda_A T_A \times e_B\lambda_B T_B} = e_C\lambda_C T_C$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (D)

Loss(copper) = gain (water + beaker)

$m_{CH}s_{CH}(T_{CH}-T) = m_W s_W (T-T_W) + m_b s_b\,(T-T_W)$

Hence final temperature can be calculated.

Q.2 (D)

Rate of melting is doubled if Rate of heat flow is doubled

and Rate $\dfrac{dQ}{dt} = \dfrac{KA(T-0)}{\ell}$

in (D) T is doubled (50 to 100°C)

and area and length are also doubled hence $\dfrac{dQ}{dt}$ doubles.

Q.3 (A, B ,C)

Heat required to melt ice

Heat given by the water = m × 10 × 1 = 10m

Heat required to melt ice > Heat given by water so complete ice will not melt.

Q.4 (A,B)

$P_1 = P_2$

$T_A\,\lambda_A = T_B\,\lambda_B$

$\sigma e_A AT_A^4 = \sigma e_B AT_B^4$
$T_A \lambda = T_B(\lambda + 1)$

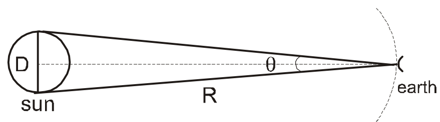
$$\frac{T_A}{T_B} = \left(\frac{0.81}{0.01}\right)^{1/4} = 3$$

$$\lambda = \frac{1}{2}\ \mu m$$

$$T_B = \frac{T_A}{3} = \frac{5802}{3} = 1934\ K$$

$\lambda_B = \lambda + 1 = 1.5\ \mu m$

Q.5 (B,C)



Let the diameter of the sun be D and its distance from the earth be R.

$$\frac{D}{R} = \theta$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore.

$$s = \frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2.$$

Thus, $s \propto T^4$ and $s \propto \theta^2$

Q.6 (A,C)

$T\lambda$ = Constant $\qquad \nu_m = \dfrac{C}{\lambda_{max}}$

$$\frac{T}{\nu_{max}} = \text{Constant}$$

$$\frac{T_1}{\nu_1} = \frac{T_2}{\nu_2} \quad \nu_2 = \frac{T_2}{T_1}.\nu_1 = \frac{2T}{T}.\nu_1 = 2\nu_1$$

$E = \sigma e A T^4$

$$E \alpha T^4 \quad \frac{E_2}{E_1} = (2)^4 = 16$$

Q.7 (C,D)
Not Reflected and Not Refracted.

Q.8 (A,B,C)
Good Absorbers are good emitters.

Q.9 (A,B,D)

$$\frac{dQ}{dt} = e A \sigma T^4$$

So, $\dfrac{dQ}{dt} \propto A$

$\propto$ e (nature of surface)
$\propto$ T (temprature)
But independent of mass.

Q.10 (A,B)

(A) $\dfrac{dQ}{dt} = e A \sigma T^4$

(Rate of emission is same initially)

(B) $\dfrac{dQ_a}{dt} = e A \sigma T_0^4$

(Rate of obsorption is same always)

(C) $\dfrac{-dT}{dt} = \dfrac{eA\sigma(T^4 - T_0^4)}{ms}$

(Due to lesser mass of hollow sphere it cools fast.) (wrong)
(D) Since hollow sphere cools fast ;
hollow will have smaller temperature at any moment.(wrong)

Q.11 (A,B,C,D)

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}\left(T^4 - T_0^4\right)$$

Q.12 (C,D)
(A) Heat absorption is surface phenomenon hence wooden (Black surface) absorbs more.(wrong)
(B) After long time both will have temperature of surroundings.(wrong)
(C) Because metal is better conductor it feels hotter.
(D) Because emission depend on surface (i.e. more for black surface)

Q.13 (AC)

$$m_A = 4m_B,\ \rho \times \frac{4}{3}\pi r_A^3 = \rho \times \frac{4}{3}\pi r_B^3 \times 4 \Rightarrow \frac{r_A}{r_B} = 4^{1/3}$$

$= 2^{2/3}$

Rate of heat loss = $\dfrac{dQ}{dt} = eA\sigma(T^4 - T_0^4)$

Ratio $\frac{(dQ/dt)_A}{(dQ/dt)_B} = \frac{A_A}{A_B} = \left(\frac{r_A}{r_B}\right)^2 = 2^{4/3}$

Rate of cooling $\frac{-dT}{dt} = \frac{dQ/dt}{ms}$

Ratio $\frac{(-dT/dt)_A}{(-dT/dt)_B} = \frac{(dQ/dt)_A}{(dQ/dt)_B} \times \frac{m_B}{m_A} = 2^{4/3} \times \frac{1}{4}$

$= 2^{-2/3}$

Q.14 (B)

In steady state $\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 4}}$

$\Rightarrow \frac{0.06 \times A \times (30-25)}{1.5 \times 10^{-2}} = \frac{0.10 \times A \times \Delta T}{3.5 \times 10^{-2}}$

$\Rightarrow \Delta T = 7°C$

$T_3 = (-10 + 7)°C = -3°C$

Q.15 (A)

$\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 3}}$

$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{0.04 \times A \times \Delta T}{2.8 \times 10^{-2}} \Rightarrow \Delta T = 14°C$

$T_3 = (-3 + 14)°C = 11°C$

Q.16 (A)

$\left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 1}} = \left.\frac{\Delta Q}{\Delta t}\right|_{\text{layer 2}}$

$\Rightarrow \frac{0.06 \times A \times 5}{1.5 \times 10^{-2}} = \frac{K_2 \times A \times 14}{1.4 \times 10^{-2}} \Rightarrow K_2 = 0.02$ W/mK

Q.17 (B)

We have $\theta - \theta_s = (\theta_0 - \theta_s)\, e^{-kt}$

where θ_0 = Initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40 to 38 in 10min, we have

$38 - 30 = (40 - 30)\, e^{-k\,10}$.... (1)

Let after 10 min, The body temp. be θ

$\theta - 30 = (38 - 30)\, e^{-k\,10}$ .... (2)

$\frac{(1)}{(2)}$ gives $\frac{8}{\theta - 30} = \frac{10}{8}$, $\theta - 30 = 6.4 \Rightarrow \theta = 36.4$ °C

Q.18 (A)

Temperature decreases exponentially.

Q.19 (C)

During heating process from 38 to 40 in 10 min. The body will lose heat in the surrounding which will be exactly equal to the heat lost when it is cooled from 40 to 38 in 10 min, which is equal to ms $\Delta\theta = 2 \times 2 = 4$ J.

During heating process heat required by the body = m s $\Delta\theta$ = 4 J.

$\therefore$ Total heat required = 8 J.

Q.20 (A,B)

(A) Emitted energy is very less for longer and shorter wavelength.

(B) From fig. at λ_m intensity is maximum

(C) Area under the curve shows amount of energy emitted.

Q.21 (A,B,C,D)

When T↑ curve shifts towards shorter wavelength hence curve spreads i.e. Area increases.

Q.22 (B)

$\lambda_m \propto \frac{1}{T}$, T' > T So, option B is correct.

Q.23 (A)

Thermal resistance is given as

$R_A = \frac{\ell}{3kA}$ $R_B = \frac{\ell}{kA}$

$\frac{R_A}{R_B} = \frac{1}{3}$

Q.24 (B)

As the rods are in series so that current is same.

$i = \frac{3k_A T_A}{\ell} = \frac{kAT_B}{\ell}$

$\frac{T_A}{T_B} = \frac{1}{3}$

Q.25 (B)

For temperature gradient comparing $\frac{dT}{dx}$ for A & B.

$i_A = i_B \Rightarrow 3kA\left(\frac{dT}{dx}\right)_A = kA\left(\frac{dT}{dx}\right)_B$

$3kAG_A = kAG_B$

$\frac{G_A}{G_B} = \frac{1}{3}$

.26 (A)

In 40 min. temperature of water has come down by 40°C. Therefore rate

$$P = \frac{mS\Delta T}{t} = \frac{0.60 \times 4200 \times 40}{40 \times 60} = 42.0\text{ W}$$

Q.27 (C)

Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\frac{m_i s_i \Delta T_i}{P} = 60$ min.

$$\Rightarrow m = \frac{(60 \times 60s) \times (42\,W)}{(2100J/kg).(30°C)} = 2.4\text{ kg}$$

Q.28 (B)

Thermal equilibrium reaches after 60 min. Ice conversion takes place for 20 min. During this time water at 0°C continues to give heat at rate P.

$$m \times L_f = P \times (20 \times 60s) \quad \Rightarrow m = \frac{42 \times 20 \times 60}{3.3 \times 10^5}\text{ kg}$$

$= 0.15$ kg

Q.29 (D)

Q.30 (B)

Q.31 (A)

(Q. 29 to 31)

As steam has comparatively large amount of heat to provide in the form of latent heat we check what amount of heat is required by the water and ice to go up to 100°C, that is

$(m_i L + m_i S_w \Delta T) + m_w . S_w . \Delta T$

$= [(200 \times 80) + (200 \times 1 \times 100)] + (200 \times 1 \times 45)$

$= 45{,}000$ cal.

That is given by m mass of steam, then

$m_s L = 45{,}000$

$$m_s = \frac{45{,}000}{540} = \frac{500}{6} = 83.3\text{ gm}$$

therefore 83.3 gm steam converts into water of 100°C.

Total water = 200 + 200 + 83.3 = 483.3 gm

steam left = 16.7 gm.

Q.32 (A) p, s, (B) t (C) q, r (D) t

(A) Initially more heat will enter through section A due to temperature difference and no heat will flow through section B because initially there is no temperature difference.

(B) At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ is same for all sections

(C) At steady state $\frac{dQ}{dt} = kA\left|\frac{dT}{dx}\right|$ or $\left|\frac{dT}{dx}\right|$

$= \frac{1}{kA}\left(\frac{dQ}{dt}\right)$

$\left|\frac{dT}{dx}\right|$ is inversely proportional to area of cross-section.

Hence is maximum at B and minimum at A

(D) At steady state heat accumulation = 0

So $\frac{dT}{dt} = 0$ for any section.

Q.33 (A) p,q, (B) r, (C) s, (D) r

(A) For a perfectly black body, both absorption and emission of radiation occurs.

(B) For a perfectly polished body cent percent reflection occurs.

(C) When radiation is incident from rarer to denser medium, both reflection and refraction occurs.

(D) When radiation is incident from denser to rarer medium, reflection always occurs but refraction may or may not occur.

NUMERICAL VALUE BASED

Q.1 [4]

$Q_{loss} = Q_{gain}$

$m_C S_C \Delta T_C = m_{ice} L$

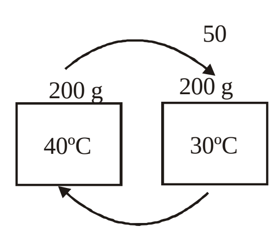
$(\rho_C A\ell)(S_C)(T - 0) = (\rho_{ice} Ah)L$

$$h = \frac{\rho_C \ell S_C T}{\rho_{ice} L}$$

$$\frac{h_A}{h_B} = \frac{\rho_A S_A}{\rho_B S_B} = \frac{(4)(0.2)}{(2)(0.1)} = 4$$

Q.2 [5]

$200 \times S \times 30 + 50 \times S \times 40 = 250 \times S \times T_1$



$T_1 = 32^o$

$32 \times S \times 50 + 150 \times S \times 40 = 200 \times S \times T_2$

$T_2 = 38^o$

$200 \times S \times 32 + 50 \times S \times 38 = 250 \times S \times T_3$

$T_3 = 33.2^o$

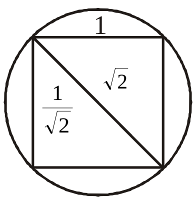
$33.2 \times S \times 50 + 150 \times S \times 38 = 200 \times S \times T_4$

$T_4 = 36.8^o$

$36.8 \times S \times 50 + 200 \times S \times 33.2 = 250 \times S \times T_5$

$T_5 = 33.92^o$

Q.3 [1716]



$$\frac{dQ}{dt} = \frac{k_1A_1}{L}\Delta T + \frac{k_2A_2}{L}\Delta T = \frac{80\times1\times78}{0.5} +$$

$$\frac{14\times\left(\pi\times\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right)\times78}{0.5}$$

$$= 156\left[80 + \frac{7(\pi-2)}{2}\right] = 156\left[80 + 7\left(\frac{22-14}{7}\right)\right]$$

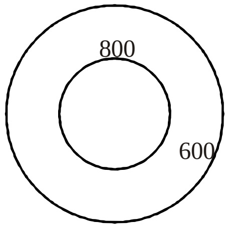
$$= 156 \times 88 \times \frac{dm}{dt} \times L_f = \frac{dm}{dt} \times 80 \times 4200$$

$$\Delta m = \frac{156\times88\times7\times60}{80\times4200} = 11 \times 156 \times 10^{-3}\text{ kg}$$

$= 1716$ gm

Q.4 [0516]

$$\frac{dQ}{dt} = \sigma \times 0.8 \times 4\pi r_1^2\,[800^4 - 600^4]$$



$$= k \times \frac{4\pi r_2 r_2'}{r_2' - r_2} \times (600 - T)$$

$r_2 r_2' \approx r_2^2$

$600 - T$

$$= \frac{\frac{17}{3}\times10^{-8}\times\frac{0.9}{10}\times\frac{1}{100}\times10^{8}\times100\times28\times10^{3}\times\frac{5}{100}}{0.085}$$

$\Rightarrow T = 516$ K

Q.5 [340]

The rate of the heat transfer is approximately proportional to the temperature difference, between radiator and room as well as between the room and the outside. The corresponding proportionality constants can be denoted as C ("radiator-room") and D("room-outside").

Then, initially,

$C(T - 300) = D(300 - 260)$

For the second set of temperatures:

$C(T - 290) = D(290 - 240)$

Solving the equations yields T = 340 K

Q.6 [2606]

The process would be

1 kg water at 80ºC $\xrightarrow{\Delta H_1}$ 1 kg water at 100ºC

$\downarrow \Delta H_2$

1 kg vapour at 100ºC and 2 atm pressure

$\Delta H_1 = ms\Delta\theta = 1 \times 4.2 \times 10^3 \times 20$

$= 8.4 \times 10^4$ J

$\Delta H_2 = mL_v + P\Delta V$

$= 1 \times 580 \times 10^3 \times 4.2 + 2 \times 10^5 \times 850.14 \times 10^{-3}$

$= 26.06 \times 10^5$ J $= 2606$ kJ

Q.7 [0041]

Neglecting other heat losses

Heat lost by water = Heat gained by thermometer

$\therefore m_1 s_1 (\theta_1 - 40^o) = m_2 s_2 (40^o - 10^o)$

m_1 = mass of water

m_2 = mass of thermometer

s_1 = specific heat of water

s_2 = specific heat of thermometer

$\Rightarrow \theta_1 = 40.6$ ºC

≈ 41ºC

Q.8 [8]

$$-\frac{dT}{dt} = \frac{K}{100 \times S_w}(T - T_0)$$

$$\int_{40}^{35} \frac{-dT}{T - T_0} = \int_0^5 \frac{K}{100 \times S_w}\,dt$$

$$\int_{40}^{35} \frac{-dT}{(T - T_0)} = \int_0^2 \frac{dt}{100 \times \rho_\ell s_\ell}$$

$$\frac{5K`}{100S_w} = \frac{2K}{100 \times \rho_\ell S_\ell}$$

$$\rho_\ell = \frac{4}{5}\ g/cm^3 = \frac{4\times10^{-3}kg}{5\times10^{-6}m^3} = \frac{4}{5}\times10^3$$

$$= \frac{40}{5}\times10^2 = 800 kg/m^3$$

Q.9 [8]

When Cu rod is used

$$\frac{100}{R_{Cu}} \times 20 = m \times L \quad .....(1)$$

when stell rod is used

$$\frac{100}{R_{stell}} \times 60 = mL \quad ....(2)$$

when both are in series

$R_{eq} = R_{Cu} \times R_{stell}$

$$\frac{100}{R_{Cu} \times R_{stell}} \times t = mL$$

from (1) & (2)

$$R_{Cu} = \frac{2000}{mL}$$

$$R_{stell} = \frac{6000}{mL}$$

$$\frac{100 \times mL \times t}{8000} = mL$$

t = 80 minutes

Q.10 **[9]**

$Q = ms\Delta T + mL$

$= 450$ cal

$450 \times 4 = 9000$ J

KVPY
PREVIOUS YEAR'S

Q.1 (A)

All the three object will be in thermal equilibrium then

% $T_1 = T_2 = T_3$

Q.2 (A)

$P = \sigma A T^4$

$= \sigma \times 4\pi R^2 \times T^4$

$$P' = \sigma \times 4\pi(2R)^2 \times \left(\frac{T}{2}\right)^4$$

$$P' = \sigma \times 4\pi R^2 T^4 \times 4 \times \frac{1}{16}$$

$$P' = \frac{P}{4}$$

Q.3 (D)

$$\frac{100 - T}{R_1} = \frac{T - 0}{R_2}$$

$$\frac{100 - T}{T} = \frac{R_1}{R_2}$$

$$R = \frac{L}{KA}$$

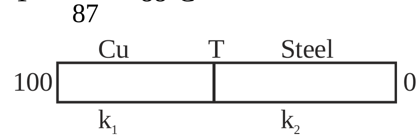
$$\frac{R_1}{R_2} = \frac{k_2}{k_1}$$

$$\frac{100 - T}{T} = \frac{50}{385} = \frac{10}{77}$$

$7700 - 77T = 10T$

$7700 = 87T$

$$T = \frac{7700}{87} = 88°C$$



Q.4 (D)

Heat loss by water = heat gain by ice.

$100 \times 1 \times 80 = m \times 80$

m = 100 gm ice melt

$\therefore$ Remaining ice = 50 g

Q.5 (B)

Ice has low thermal conductivity

So no exchange of heat outside surrounding.

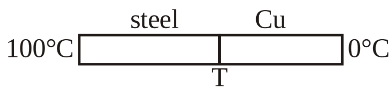
Q.6 (B)

Energy radiated, $U \propto AT^4 t$

$$\Rightarrow \frac{U_2}{U_1} = \frac{A/2(2T)^4.t}{AT^4.t} = 8$$

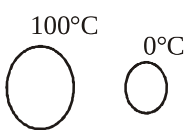
$\Rightarrow U_2 = 8U_1$
$\Rightarrow mS\Delta t_2 = 8mS\Delta t_1$
$\Rightarrow \Delta t_2 = 8\Delta t_1$

Q.7 (A)



$$\frac{50 \times A \times (100 - T)}{0.1} = \frac{400 \times A \times (T - 0)}{0.2}$$

Q.8 (C)



$\because$ Heat capacity increase with temperature

Q.9 (A)

$Pt = m_w S_w \Delta T + m_c s_c \Delta T$
$10 \times 15 \times 60 = 0.5 \times 4200 \times 3 + m_c s_c \times 3$
$9000 = 6300 + m_c s_c 3$
$m_c s_c = 900$ J/k.
Now, for oil
$10 \times 20 \times 60 = 2 \times S_0 \times 2 + 900 \times 2$
$12000 - 1800 = 4 S_0$

$$S_0 = \frac{10200}{4} = 2.51 \times 10^3 \text{ J/kg} - k$$

Q.10 (D)

$$3\frac{kA}{\ell}[100 - T] = \frac{kA}{\ell}[T - 0]$$

$3w - 3T = T$
$T = 75^o$ C

Q.11 (A)

E Radiated by Sun
$E = 4\pi r^2 \times 1.4$ kW $= mC^2$
$E = 4\pi \times (1.5\,5\,10^{11}) \times 1.4 \times 10^3 = m.(3 \times 10^8)^2$

$$m = \frac{4 \times 22 \times (1.5)^2 \times 1.4 \times 10^9}{7 \times 9} = 10^9 \text{ kg/s}$$

Q.12 (C)

Specific heat of water is very high
$\therefore$ It temperature rises by small amount.

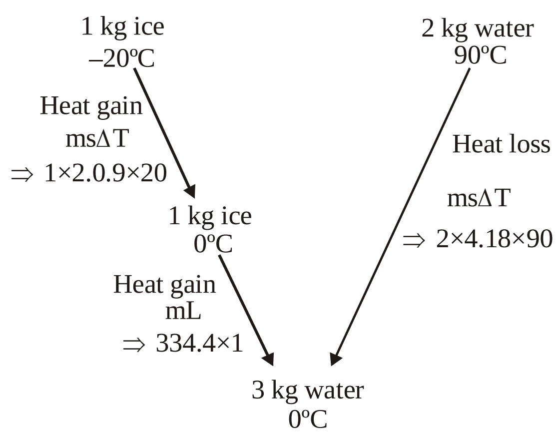
Q.13 (D)

Surface area of Ice get increases by crushing and colling due to ice occur due to convection process which is proportional to area.

Q.14 (C)

Calorimetry principle
Heat lost = Heat gain
Heat loss by aluminum = Heat gain by water
$50 \times 10^{-3} \times 900 \times (300 - 160) = 1 \times 4200 \times (T - 30)$
$\Rightarrow 6300 = 4200\,(T - 30)$
$\Rightarrow 1.5 = T - 30$
$\Rightarrow T = 31.5^oC$

Q.15 (A)



Total heat gain $= 20 \times 2.09 + 334.4$ KJ $= 376.2$ kJ
Total heat loss $= 752.4$ kJ
Heat gain required $= 752.4 - 376.2 = 376.2$ kJ
$376.2 = ms\Delta T$
$376.2 = 3 \times 4.18 \times \Delta T$
$\Delta T = 30$ centigrate
$T_{final} = 30^oC$

Q.16 (A)

Because on earth there is no atmosphere. So water will boil. (At Boiling point vapour pressure = Atmospheric pressure, in open vessel)

Q.17 (C)

$MB = 20 \times 10^{-3}$ Kg
$CB = 5000$ J / Kg-ºC
$V = 2000$ M/s

$M_w = 1$ Kg
$C_w = 3000$ J / Kg – °C
$T_f = 25°$ C = 298 K

$$\frac{1}{2}MV^2 = M_w C_w \Delta T_w + M_B C_B \Delta T_B$$

$$= \frac{1}{2}M_B V^2 = M_w C_w (\Delta T_w) + M_B C_B \Delta T_B$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times 4 \times 10^6$$

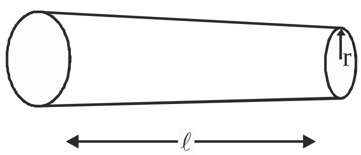
$$= (\Delta T)\{1\times3000 + 20 \times 10^{-3} \times 5000\}$$

$\Rightarrow 40 \times 10^3 = \Delta T \{3000 + 100\}$

$$\Delta T = \frac{40\times10^3}{3100}$$

$\Delta T = 12.9$
$T_f - 25 = 12.9$
$T_f = 25 + 12.9 = 37.9°C$

Q.18 (B)



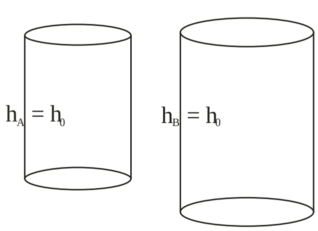
$$\frac{dm}{dt}\times S\Delta T = \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 5\times10^8 \times \text{volume of rod}$$

$$= 5\times10^8\times\pi\times(4)^2\times10^{-6}\times\frac{0.2}{10}$$

$= 5 \times 10 \times \pi \times 16 \times 2$
$= 1600\pi$
$0.2 \times 4\times 10^3 \Delta T = 1600\pi$
$8 \times 10^2 \Delta T = 16 \times 10^2 \pi$
$\Delta T = 3.14 \times 2$
$\Rightarrow 6.28°C$

Q.19 (B)



$$t \propto \frac{m}{A}$$

$$t = k.\frac{V_\rho}{A} = k.\rho h$$

$t \propto h$

$$\therefore \frac{t_A}{t_B} = \frac{h_A}{h_B} = \frac{h_0}{2h_0} \Rightarrow t_B = 2t_A$$

Q.20 (B)

$$ds = ds_1 + ds_2$$

$$ds = \frac{-Q}{T_1} + \frac{Q}{T_2}$$

$$ds = -Q\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\frac{ds}{dt} = -\frac{Q}{dt}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\frac{ds}{dt} = \frac{-(T_1 - T_2)}{R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$$

$$\frac{ds}{dt} = \frac{(T_1 - T_2)}{R}\left[\frac{T_1 - T_2}{T_1T_2}\right]$$

$$\frac{ds}{dt} = \left(\frac{T_1^2 + T_2^2 - 2T_1T_2}{T_1T_2}\right)\frac{1}{R}$$

$$\frac{ds}{dt} = \left(\frac{T_1}{T_2} + \frac{T_2}{T_1} - 2\right)\frac{1}{R}$$

$$y = \left(x + \frac{1}{x} - 2\right)\frac{1}{R}$$

Q.21 (C)

Vaporization rate of water = 20 g/h
Water vaporized in 2 hour = 20×2 gm

$$dm = \frac{40}{1000}\,kg$$

$$\frac{\text{Latent heat of vaparisation}}{\text{specific heat of water}} = 540 = \frac{L}{C}$$

heat contain in vaporised vapor = (dm).L = (dm). L
Heat lost by water in earthen pitcher = mc. dT
m = 4kg
heat loss by water in earthen pitcher = heat contain in vaporised water dm. L = m.C. dT

$$\frac{40}{1000}\left(\frac{L}{C}\right) = 4.\ dT$$

$$dT = \frac{1}{100} \times 540 = 5.4\ ºC$$

$dT = 5.4\ ºC$

Q.22 (A)

$100º\ L = 200\ ºC$
$1º\ L\ = 2\ º\ L$
$0\ ºC = 25ºL$
$100\ ºC = 75ºL,$

Q.23 (B)

$50\ t_0\ (540) + 50\ to\ (100 - 70) = 500\ (1)\ (70 - 25)$
$28500\ t_0 = 22500$
$t_0 = 0.789\ min = 47\ sec$

Q.24 (D)

The minimum observed intensity of the parent star is $0.9999\ I_0$

Q.25 (D)

$P_{body} = \sigma(4\pi(50R_s)^2)(2T_s)^4$
$\Rightarrow\ P_{body} = P_{sun} \times (50)^2 \times (2)^4$
Intensity at earth due to body

$$= \frac{P_{body}}{4\pi R^2_{body}} = \frac{(50)^2 \times 2^4 \times P_{sun}}{4 \times 10^{20} \times (4\pi AU)^2}$$

$\Rightarrow\ I_{body} = 10^{-16} \times I_{sun}$

Q.26 (B)

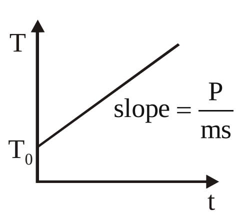
$$P = ms\frac{dT}{dt}$$

$$\Rightarrow \int_0^1 Pdt = \int_{T_0}^{T} msdT$$

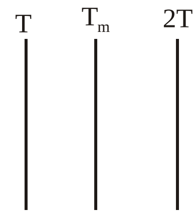
$$= Pt = ms\ (T - T_0)$$

$$\Rightarrow\ T = \frac{P}{ms}t + T_0$$

where T_0 is temperature at $t = t_0$



Q.27 (C)



Rate of heat absorbed = Rate of heat emitted
$\sigma AT^4 + \sigma A(2T)^4 = \sigma 2atM^4$

$$T_m = \left(\frac{17}{2}\right) = 1.7\ T$$

Q.28 (A)

$$\frac{\Delta\ell}{\ell} = \alpha\Delta\theta = \frac{F}{yA}$$

$$\Delta\theta = \frac{F}{yA\alpha} = 5°C$$

$\Delta\theta = 20 - T = 5$
$T = 15°C$

Q.29 (B)

$$\frac{dQ}{dt} = eA\sigma[T_0^4 - T_s^4]$$

$e = 1, A = 7 \times 10^{-2}, s = 5.67 \times 10^{-8}$
$T_0 = 333\ K, T_s = 273\ K$

$$\frac{dQ}{dt} = 26.75\ \text{Watt}$$

total energy produced $= \frac{10}{100} \times 30 \times 10^3 \times 300$

$\Rightarrow 9 \times 10^5\ J$

$$\therefore\ \ \text{time} = \frac{9 \times 10^5}{26.75 \times 3600}\text{hrs} = 9.35\ \text{hrs}$$

JEE-MAIN PREVIOUS YEAR'S

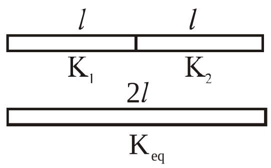
Q.1 (4)

Sol. $$\frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$\theta_2 - \theta R_1 = \theta R_2 - \theta_1 R_2$
$\theta[R_1 + R_2] = \theta_1 R_2 + \theta_2 R_1$

$$\theta = \frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$$

Q.2 (1)



$$R_{eff} = \frac{l}{K_1A} + \frac{l}{K_2A} = \frac{2l}{K_{eq}A}$$

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

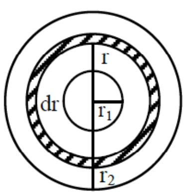
Q.3 (3)

Q.4 (2)

Q.5 [57]

Q.6 (4)

Q.7 (1)



Thermal resistance of spherical sheet of thickness dr and radius r is

$$dR = \frac{dr}{K(4\pi r^2)}$$

$$R = \int_{r_1}^{r_2} \frac{dr}{K(4\pi r^2)}$$

$$R = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi K}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Thermal current (i) = $\dfrac{\theta_2 - \theta_1}{R}$

$$i = \frac{4\pi K r_1 r_2}{r_2 - r_1}(\theta_2 - \theta_1)$$

Q.8 [2]

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 (A,C,D)

A : At steady state, heat flow through A and E are same.

C : $\Delta T = i \times R$

'i' is same for A and E but R is smallest for E.

D : $i_B = \dfrac{\Delta T}{R_B}$

$i_C = \dfrac{\Delta T}{R_C}$

$i_D = \dfrac{\Delta T}{R_D}$

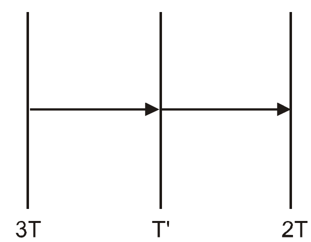
if $i_c = i_B + i_D$

Hence $\dfrac{1}{R_C} = \dfrac{1}{R_B} + \dfrac{1}{R_D}$

$\Rightarrow \dfrac{8KA}{\ell} = \dfrac{3KA}{\ell} + \dfrac{5kA}{\ell}$

Q.2 (C)

In steady state energy absorbed by middle plate is equal to energy



released by middle plate.

$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$

$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$

$(2T')^4 = (16 + 81)\,T^4$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

Q.3 (A)

In configuration 1 equivalent thermal resistance is $\dfrac{3R}{2}$

In configuration 2 equivalent thermal resistance is $\dfrac{R}{3}$

Thermal Resistance $\propto$ time taken by heat flow from high temperature to low temperature

Q.4 (A)

In steady state

$I\pi R^2 = \sigma(T^4 - T_0^4)\,4\pi R^2 \quad \Rightarrow \quad I = \sigma\,(T^4 - T_0^4)\,4$

$\Rightarrow\ T^4 - T_0^4 = 40 \times 10^8 \quad \Rightarrow \quad T^4 - 81\times10^8 = 40 \times10^8$

$\Rightarrow\ T^4 = 121 \times 10^8 \quad \Rightarrow \quad T \approx 330\ K$

Q.5 (B)

$$P_{\text{device}} - P_{\text{cooler}} = \frac{ms\Delta T}{\Delta t}$$

$$3000 - P = \frac{120\times4.2\times10^3\times20^0}{3\times60\times60}$$

$\Rightarrow P = 2067W$

Hence, (B)

Q.6 (C,D)

As $\lambda_m T$ = constant

& $P = \dfrac{V^2}{R}$

Hence, (c, d)

Q.7 [9]

$$1 = \log_2\left\{\frac{eA\sigma(487+273)^4}{P_0}\right\}$$

$$\& \ x = \log_2\left\{\frac{eA\sigma(2767+273)^4}{P_0}\right\}$$

$\therefore x = 9$

Q.8 (A)

$$\frac{T_Q - 10}{\frac{R}{2}} = \frac{400 - T_Q}{R} \qquad \Rightarrow T_Q = 140$$

$$\frac{dl}{dx} = \alpha\Delta T$$

$$\int_0^{\Delta l} dl = \int_0^1 \alpha(130x)\,dx \qquad \Rightarrow \Delta l = 0.78mm$$

Hence, (A)

Q.9 (B)

(A) Since the temperature of the body remains same, therefore heat rdiated by the body is same as before.

$\left(W_1 = \sigma a T^4 = \sigma a(310)^4\right)$

(B) $W \propto$ Area

If exposed area deacreases, energy radiated also decreases.

(C) $\lambda_m T = b \quad \Rightarrow \quad T\uparrow,\ \lambda_m\downarrow$

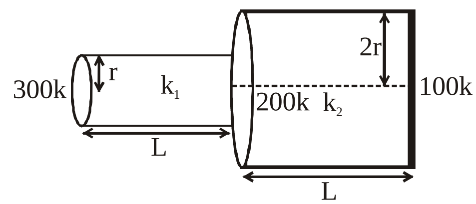
(D) $(W_1 = \sigma a T^4 = \sigma a(310)^4)$

$$\sigma T_0^4 = 460\,Wm^{-2}$$

$\sigma a(310)^4 > 460\ Wm^{-2}$

Q.10 [4.00]

We have in steady state,



$$\left(\frac{\frac{200-300}{L}}{k_1\pi r^2}\right) + \left(\frac{\frac{200-100}{L}}{k_2\pi(2r)^2}\right) = 0$$

$$\Rightarrow \frac{k_1\pi r^2\times100}{L} = \frac{100k_2\pi\times4r^2}{L} \Rightarrow \frac{k_1}{k_2} = 4$$

Q.11 (A)

$P = \dfrac{dQ}{dt} \qquad T_{(t)} = T_0\,(1+\beta t^{1/4})$

$$\frac{dQ}{dt} = \boxed{ms}\,\frac{dT}{dt} \Rightarrow S = \frac{P}{\left(\frac{dT}{dt}\right)}$$

$$\frac{dT}{dt} = T_0\left[0+\beta\frac{1}{4}.t^{-3/4}\right] = \frac{\beta T_0}{4}.t^{-3/4}$$

$$S = \frac{P}{(dT/dt)} = \frac{4P}{\beta T_0}.t^{3/4}$$

$$S = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right]$$

$$\frac{T(t)}{T_0} = (1+\beta t^{1/4})$$

$$\beta t^{1/4} = \frac{T(t)}{T_0} - 1 = \frac{T(t)-T_0}{T_0}$$

$$t^{3/4} = \left(\frac{T(t)-T_0}{\beta.T_0}\right)^3$$

$$\Rightarrow S = \frac{4P}{T_0\beta}\left[\frac{T(t)-T_0}{\beta.T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}[T(t)-T_0]^3$$

Q.12 [270.00]

Let m = mass of calorimeter,
x = specific heat of calorimeter
s = specifc heat of liquid
L = latent heat of liquid
First 5 g of liquid at 30° is poured to calorimter at 110°C

$\therefore\ m \times x \times (110-80) = 5 \times s \times (80 \times 30) + 5\,L$

$\Rightarrow mx \times 30 = 250\,s + 5\,L$ ... (i)

Now, 80 g of liquid at 30° is poured into calorimeter at 80°C, the equilibrium temperature reaches to 50°C.

$\therefore\ m \times x \times (80-30) = 80 \times s \times (50-30)$

$\Rightarrow mx \times 30 = 1600\,s$ .... (ii)

From (i) & (ii)

$250\,s + 5\,L = 1600\,s \Rightarrow 5L = 1350\,s$

$$\Rightarrow \frac{L}{s} = 270$$

Q.13 (B, C, D)

$A=64mm^2$, $T=2500$ K (A=surface area of filament, T=temperature of filament, d is distance of bulb from observer, R_e=radius of pupil of eye)

Point source d = 100 m

R_e=3mm

(A) $P=\sigma AeT^4$

$=5.67\times10^{-8}\times64\times10^{-6}\times1\times(2500)^4$ (e=1 black body)

= 141.75 w

Option (A) is wrong

(B) Power reaching to the eye

$$= \frac{P}{4\pi d^2}\times\left(\pi R_e^2\right)$$

$$= \frac{141.75}{4\pi\times(100)^2}\times\pi\times\left(3\times10^{-3}\right)^2$$

$= 3.189375\times10^{-8}$W

Option (B) is correct

(C) $\lambda_m T=b$

$\lambda_m\times2500=2.9\times10^{-3}$

$\Rightarrow \lambda_m = 1.16\times10^{-6}$

= 1160 nm

Option (C) is correct

(D) Power received by one eye of observer

$$= \left(\frac{hc}{\lambda}\right)\times N$$

N=Number of photons entering into eye per second

$\Rightarrow 3.189375\times10^{-8}$

$$= \frac{6.63\times10^{-34}\times3\times10^8}{1740\times10^{-9}}\times N$$

$\Rightarrow N=2.79\times10^{11}$

Option (D) is correct

Q.14 (8.33)

$$\frac{dQ}{dt} = \sigma eA\left(T^4 - T_0^4\right) \quad \text{...(i)}$$

$$\frac{dQ}{Adt} = e\sigma\left(T_0+\Delta T\right)^4 - T_0^4\big) = \sigma T_4^0\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= e\sigma T_0^4\left[\left(1+4\frac{\Delta T}{T_0}\right)-1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3\cdot 4\Delta T \quad \text{...(ii)}$$

Now from equ. (i)

$$ms\frac{dT}{dt} = \sigma eT\left(T^4 - T_0^4\right)$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}\left[\left(T_0+\Delta T\right)^4 - T_0^4\right]$$

$$= \frac{\sigma eA}{ms}T_0^4\times\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}T_0^4.4\Delta T$$

$$\frac{dT}{dt} = e\Delta T;\left(K = \frac{4\sigma eAT_0^3}{ms}\right)$$

$$\Rightarrow 4\sigma eAT_0^3 = \frac{K}{A}(ms)$$

from equ. (i)

$$\frac{dQ}{Adt} = e\sigma T_0^3.4\Delta T$$

700=(K/A)(ms) ΔT

$$\therefore \Delta T = \frac{700\times5\times10^{-2}}{10^{-3}\times4200} = \frac{50}{6} = \frac{25}{3}$$

$\Delta T=8.33$

Q.15 (9)

Kinetic Theory of Gases and Thermodynamics

EXERCISES-I

ELEMENTARY

Q.1 (1)

$$V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{200}{V_2} = \frac{(273+20)}{(273-20)} = \frac{293}{253}$$

$$V_2 = \frac{200 \times 253}{293} = 172.6\,ml$$

Q.2 (2)

$\vec{P} = M\vec{V}av$, As $\vec{V}av = 0$ (in equilibrium)

$\therefore \vec{P}_{av} = 0$

Q.3 (2)
The collision of molecules of ideal gas is elastic collision

Q.4 (3)
one molecule has some single value of speed which is equal average speed and rms speed of the gas
$\therefore V_a = V_{rms}$.

Q.5 (2)

$$V_{av} \,\alpha\, \frac{1}{\sqrt{M_0}}$$

$\therefore$ oxygen molecule hits the wall with smaller average speed

Q.6 (2)

$$V_{av} = \sqrt{\frac{8RT}{\pi M_0}}, V_{AV} \,\alpha\, \sqrt{T}$$

For same temp in vessel A, B and C, Average speed of O_2 molecule is same in vessel A and C and is equal to V_1.

Q.7 (2)
As ΔU is a state function i.e., it depends initial and final position in process A and B initial and final temp are same.
$\therefore \Delta U, = \Delta U_2$.

Q.8 (2)

$$T_2 = \left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{1.5V}{V}\right) \times (273+27) = 450\,K$$

$\Rightarrow 177°C$

Q.9 (4)

$$v_{rms} = \sqrt{\frac{3kT}{m}} = v_{rms} \propto \frac{1}{\sqrt{m}}$$

Q.10 (2)

$v_{rms} \propto \sqrt{T}$

Q.11 (3)

$$V = \sqrt{\frac{3RT}{M_0}}$$

Q.12 (3)

$$V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V}{2V} = \frac{(273+27)}{T_2} = \frac{300}{T_2}$$

$T_2 = 600\,K = 327°C$

Q.13 (1)
$V \propto T$ (as constant pressure)

Q.14 (3)
Boyle's and Charle's law follows kinetic theory of gases

Q.15 (1)
A monoatomic gas molecule has only three translational degree of freedom.

Q.16 (3)
A diatomic molecule has three translational and two rotational degree of freedom.
Hence t otal degree of freedom f = 3 + 2 = 5

Q.17 (1)

$$\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{f}$$

Q.18 (3)
As compare to gas solid expand very less.
$\therefore C_p$ is slightly greater then C_v.

Q.19 (1)
$(\Delta Q)_v = \mu C_v\, \Delta T \Rightarrow (\Delta Q)_v = 1 \times C_v \times 1 = C_v$

For monoatomic gas $C_v = \frac{3}{2}R \Rightarrow (\Delta Q)_v = \frac{3}{2}R$

Q.20 (4)
Kinetic energy is function of temperature

Q.21 (1)
For cyclic process. Total work done $= W_{AB} + W_{BC} + W_{CA}$
$\Delta W_{AB} = P\Delta V = 10\,(2-1) = 10$ J and $\Delta W_{BC} = 0$
(as V = constant)
From FLOT, $\Delta Q = \Delta U + \Delta W$
$\Delta U = 0$ (Processs ABCA is cyclic)
$\Rightarrow \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$
$\Rightarrow 5 = 10 + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5$ J

Q.22 (1)
$T \alpha P$ or $\frac{P}{T}$ = constant
As $\frac{P}{T} = \frac{nR}{V}$ = constant or V = constant
$\therefore$ W = 0.

Q.23 (4)
Work done = Area under curve $= \frac{6P_1 \times 3V_1}{2} = 9P_1V_1$

Q.24 (2)
As Volume decreases
$\therefore$ pressure of the gas in the cylinder increases

Q.25 (1)
$\Delta Q = AU + \Delta W$ and $\Delta W = P\Delta V$

Q.26 (2)
In process AB
T = constant
P = increases $P \,\alpha\, \frac{1}{V}$
or V = decreases $\Delta Q = \Delta W$.
$\Delta W = -$ ve. or
$\Delta Q = -$ ve
$\therefore$ heat is rejected out of the system

Q.27 (2)
$\Delta Q = \Delta W$
(T = constant)
if heat is supplied then $\Delta W = +$ve

Q.28 (2)
$B \rightarrow A$
$\Delta Q = 0$
$0 = -30 + \Delta U_{BA}$
$\Delta U_{BA} = 30$ J
$\therefore \Delta U_{AB} = -\Delta U_{BA} = -30$ J

Q.29 (2)
$\Delta Q = \Delta U + \Delta W$; $\Delta Q = 200$ J and $\Delta W = -100$ J
$\Rightarrow \Delta U = \Delta Q - \Delta W = 200 - (-100) = 300$ J

Q.30 (4)
Heat given $\Delta Q = 20$ ca*l* $= 20 \times 4.2 = 84$ J.
Work done $\Delta W = -50$ J
[As process is anticlockwise]
By first law of thermodynamics $\Rightarrow \Delta U = \Delta Q - \Delta W = 84 - (-50) = 134$ J

Q.31 (3)
In isothermal process temperature remains constant.

Q.32 (1)
For free expansion
Q = 0, W = 0, $\Delta U = 0$

Q.33 (3)

Q.34 (1)
$E_\theta = P$

Q.35 (3)
For free expansion
$\Delta U = 0$ or $\Delta T = 0$
$\therefore$ U or T = const

Q.36 (3)
Adiabatic process
$\Delta Q = 0$
For any process
$\Delta_V = nC_V\Delta T$
Hence, option (3) is correct.

Q.37 (1)
Work done

Q.38 (1)
Initial and final states are same in all the process.
Hence $\Delta U = 0$; in each case.
By FLOT; $\Delta Q = \Delta W$ = Area enclosed by curve with volume axis.
$\because$ (Area)$_1$ < (Area)$_2$ < (Area)$_1 \Rightarrow Q_1 < Q_2 < Q_3$- .

Q.39 (1)
$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = \frac{1}{4} = 25\%$$
So 26 % efficiency is impossibel

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (3)

$V_{av} = \sqrt{\frac{8KT}{\pi m}}$, as T = constant $\therefore V_{av}$ = constant

Q.2 (4)

$\vec{P}_{av} = M\vec{V}_{av}$, as the average momentum of an ideal gas is zero

$\therefore$ option D is correct.

Q.3 (1)

$$\sqrt{\frac{3RT}{32}} = \sqrt{\frac{3R \times 273}{28}}$$

$$T = \frac{273 \times 32}{28} = 426.3 \text{ k.}$$

Q.4 (2)

Real gas behaves as an ideal gas at low pressure and high temperature

Q.5 (3)

one molecule has some single value of speed which is equal most probabla speed and average speed of the gas

$\therefore \quad V_{mp} = V_{av}$.

Q.6 (3)

$$V_{AV} = \sqrt{\frac{8RT}{\pi M_0}} = v$$

for nitrogen $V_{AV} = \sqrt{\frac{8R \times 2T}{\pi M_0 / 2}} = v$.

for nitrogen

$$V_{AV} = \sqrt{\frac{8R \times 2T}{\pi M_0 / 2}} = v .$$

Q.7 (2)

$$V_{av} \, \alpha \, \frac{1}{\sqrt{M_0}}$$

$\therefore$ oxygen molecule hits the wall with smaller average speed

Q.8 (2)

$$V_{av} = \sqrt{\frac{8RT}{\pi M_0}} , V_{AV} \, \alpha \, \sqrt{T}$$

For same temp in vessel A, B and C, Average speed of O_2 molecule is same in vessel A and C and is equal to V_1.

Q.9 (1)

As translation K.E is = $\frac{3}{2}$nRT

$$E = \frac{3}{2} PV$$

where E = total translational K.E.

Q.10 (3)

For an ideal gas, the no of molecules of equal moles of gas is same .

Q.11 (3)

From the formula

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$V_{rms_{o_2}} = \sqrt{\frac{3RT_{o_2}}{M_{o_2}}} \Rightarrow$$

$$V_{rms\,o} = \sqrt{\frac{3RT_{o_2} \times 2}{M_{o_2}/2}} = 2\sqrt{\frac{3RT_{o_2}}{M_{o_2}}} = 2V$$

Q.12 (2)

The average velocity is given as

$$V_{av} = \sqrt{\frac{8RT}{\pi M}}$$

Independent of other gases. Hence average velocity of oxigen in third container will be V_1 only.

= 7.66 u

Q.13 (4)

$$V_{avg} = \frac{1+2+3.........N}{N} = \frac{N(N+1)}{2N} = \frac{(N+1)}{2}$$

$$V_{rms} = \sqrt{\frac{1^2 + 2^2 + ...N^2}{N}} = \sqrt{\frac{2N+1}{6}}$$

$$\frac{V_{rms}}{V_{avg}} = \frac{2}{(N+1)}\sqrt{\frac{2N+1}{3}}$$

Q.14 (1)

Average rotational K. E. = $\frac{1}{2}KT \times 2 = KT$

So it will be same for both the gases.

Q.15 (1)

We are given $P = \frac{2E}{3V}$.

$PV = \frac{2}{3}E$

$E = \frac{3}{2}n\,RT$.

Here E is the Translational K.E. for all the particles.

Q.16 (1)

We know that

$PV = nRT$

$$n = \frac{PV}{RT} = \frac{1.3 \times 10^5 \times \left[7 \times \left(10^{-2}\right)^3 \times 10^3\right]}{8.3 \times 273}$$

So, Number of molecules is

$$= \frac{1.3 \times 10^5 \times 7 \times 10^{-3}}{8.3 \times 273} \times 6.023 \times 10^{23} = 2.4 \times 10^{23}$$

Q.17 (1)

$\frac{Pm}{\rho} = nRT$

slope of T_1 > slope of T_2

$\therefore \quad T_1 > T_2$

Q.18 (3)

$PV = nRT$

$\therefore$ temperature remains same for all ideal gas

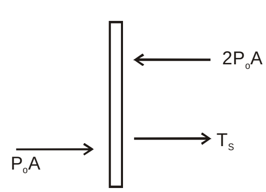
Q.19 (3)

As the volume remains constant on increasing temperature pressure becomes double.

V = const.

T = doubled

$p = 2P_o$



$\Rightarrow T_S + P_oA = 2P_oA$

$T_S = P_oA$

Q.20 (4)

$U = \frac{nfRT}{2} = \frac{nfN_AkT}{2}$

$\frac{2U}{fkT} = nN_A = N$

Q.21 (2)

As ΔU is a state function i.e., it depends initial and final position

in process A and B initial and final temp are same.

$\therefore \quad \Delta U_1 = \Delta U_2$.

Q.22 (4)

As $\Delta U = nR\Delta T$

For closed path

$\Delta T = 0$

$\therefore \quad \Delta U = 0$.

Q.23 (4)

As $C_p - C_v = R$

For above equation, we can say that both C_p and C_v increase by same amount.

Q.24 (3)

$s = \frac{Q}{m\Delta T}$

For changing state

T = const or $\Delta T = 0$

$\therefore s = \infty$ (infinite)

Q.25 (1)

As f = 5

$dU = nC_v\,dT = \frac{nfRdT}{2}$

$C_v = \frac{fR}{2}$

$\therefore C_v = \frac{5R}{2}$

Q.26 (3)

Gas has different specific heat for different processes

$\therefore$ gas has infinite number of specific heats.

Q.27 (3)

$\Delta U > 0$

and $\Delta W > 0$

$\therefore C > C_v$

Q.28 (3)

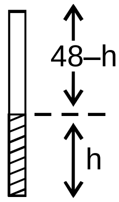
As compare to gas solid expand very less.

$\therefore$ C_p is slightly greater then C_v.

Q.29 (3)

In the final condition.

Let atmospheric pressure is P and ht of liquid column is h.



$$P + h = 76$$
$$P_1V_1 = P_2V_2$$
$$76 \times 5 = P(43 - h)$$

$380 = (76 - h)(43 - h)$
$h = 38$ cm
So, $48 - h = 10$ cm $= 0.1$ m.

Q.30 (1)
$P + 50 = 75$
$P = 25$ cm of H_g

$$\frac{10^5}{75} \times 25$$
$= 33.3$ kPa

Q.31 (1)
$$\frac{1}{2}mv^2 = nC_v dT$$
$$\frac{1}{2}mv^2 = \frac{m}{.03}\left(\frac{5}{2}R\right)\Delta T$$
$$\Delta T = \frac{.03 \times 100^4}{5R} = \frac{6 \times 10^{-3} \times 10^4}{R} = \frac{60}{R}$$

Q.32 (1)
T α P or $\frac{P}{T}$ = constant

As $\frac{P}{T} = \frac{nR}{V}$ = constant or V = constant

$\therefore$ W = 0.

Q.33 (4)
work done on the gas = negative work
W = PdV
when V $\rightarrow$ decreases
then W = – ve
hence option D is correct.

Q.34 (1)
As volume increases
$\therefore$ WD continuously increases

Q.35 (3)
As $W = P\Delta V$
ΔV = same is both process
As $P_B > P_A$
$\therefore$ $\Delta W_2 > W_1$.
As $P_B > P_A$
$\therefore$ $\Delta W_2 > W_1$.

Q.36 (2)
$$W = \int_{V_1}^{V_2} PdV$$
$$= \int_{V_1}^{V_2} aV^2 dV = a\left[\frac{V^3}{3}\right]_{V_1}^{V_2} = \frac{a}{3}\left(V_2^3 - V_1^3\right)$$
$$= \frac{1}{3}\left[P_2V_2 - P_1V_1\right] = \frac{1}{3}\left[nR\Delta T\right] = \frac{1}{3}R\left(T_2 - T_1\right)$$

Q.37 (1)

Q.38 (4)
ΔU = same is both process
$Q_{acb} - W_{acb} = Q_{adb} - W_{adb}$.
$200 - 80 = 144 - W_{adb}$.
$W_{adb} = 24$ J.

Q.39 (2)
$\Delta U = Q_{acb} - W_{acb} = 200 - 80 = 120$ J
$\Delta U = Q_{ba} - W_{ba}$, $-120 = Q_{ba} + 52$, $Q_{ba} = -172$ J.

Q.40 (4)
$U_b - U_a = 120$
$U_b = 120 + 40 = 160$J

Q.41 (2)
in db.
$W_{db} = 0$
$U_b - U_d = Q_{db}$.
$160 - 88 = Q_{db}$

$Q_{db} = 72$J.

Q.42 (2)
$B \rightarrow A$
$\Delta Q = 0$
$0 = -30 + \Delta U_{BA}$
$\Delta U_{BA} = 30$ J
$\therefore$ $\Delta U_{AB} = -\Delta U_{BA} = -30$ J

Q.43 (2)
$\Delta Q = \Delta U + \Delta Q$
$\Delta U = \Delta Q - \Delta W$

$\Delta U = Q - P_o \Delta V$

$\Delta U = Q - P_o\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$

Q.44 (4)

$W_{net} = W_{1-2} + W_{2\to3} + W_{3-1}$

$10 = W_{1-2} + 0 - 20$

$W_{1-2} = 30\,J$

$\Delta U_{1-2} = 0$

$\therefore\ \Delta Q_{1-2} = \Delta W_{1-2} + \Delta U_{1\to2} = 30\,J$

Q.45 (1)

$\Delta Q = \Delta W + 3\Delta W$

$= 4\Delta W$

$\because\ n = \frac{\Delta W}{\Delta Q} = \frac{\Delta W}{4\Delta W} = 0.25$

Q.46 (1)

Free Expansion

So, $\left.\begin{array}{l}\Delta W = 0\\ \Delta Q = 0\end{array}\right] \Rightarrow \Delta U = 0 \quad \Rightarrow \Delta T = 0$

and $P_1V_1 = P_2(2V_1)$

$P_2 = \frac{P_1}{2}$

Q.47 (4)

I[st] Process

$\Delta U_1 = \Delta Q_1 - \Delta W_1$

$= 16 - 20 = -4\,KJ$

II[nd] Process

$\Delta W_2 = \Delta Q_2 - \Delta U_2$

$\Delta U_1 = \Delta U_2 \qquad (\therefore \Delta T = \text{same})$

So, $\Delta W_2 = [9 - (-4)] = 13\,KJ$

Q.48 (3)

Q.49 (2)

As PV = nRT $m = \rho V$ = constant or $\rho\ \alpha\ \frac{1}{V}$ and $P\ \alpha\ \rho$

$A \to B$ T = constant, pressure increases or volume decreases

$B \to C$ Volume is constant, V = constant

$C \to D$ P is decreases or volume increases [T = constant]

$D \to A$ Volume is constant V = with constant, clearly option 'B' is constant.

Q.50 (2)

$\Delta U = 0$

$\therefore$ T = constant

or PV = constant or P-V curve is a rectangular hyperbola.

clearly, option B is correct.

Q.51 (3)

$\frac{V}{T} = \frac{nR}{P}$

$\frac{1}{P}\ \alpha$ slope or $P\ \alpha\ \frac{1}{\text{slope}}$

$\therefore\ P_2 < P_1$

Q.52 (4)

In isothermal expansion

T = constant $\Delta U = 0$

$W = \Delta Q$

$\therefore$ option (4) is correct.

Q.53 (3)

W.D. = π × Pressure Radius × volume Radius (area of ellipse)

$W = \pi\left(\frac{P_2 - P_1}{2}\right)\left(\frac{V_2 - V_1}{2}\right) = \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$

Q.54 (2)

$L \to M$

P = constant

$V\ \alpha\ T$.

MN T = constant

Here, option B is constant

Q.55 (1)

As initial and final state are same

$\therefore\ T_I = T_F$ As V_{rms}, $\vec{P}_{av}$ and $\vec{K}_{av}$ depends on temperature

$\therefore$ all are equal.

Q.56 (2)

In process AB

T = constant

P = increases $P\ \alpha\ \frac{1}{V}$

or V = decreases $\Delta Q = \Delta W$.

$\Delta W = -$ ve. or $\Delta Q = -$ ve

$\therefore$ heat is rejected out of the system.

Q.57 (3)

$\Delta Q = \Delta W$ (T = constant) if heat is released then

$W = -$ve

Q.58 (4)

dQ = dW + dU

dQ = PdV + dU

dQ = nRdT + dU

$$dQ = \frac{2dU}{f} + dU \left[dU = \frac{nfRdT}{2}\right]$$

$$\frac{dU}{dQ} = \frac{1}{\left(\frac{2}{f}+1\right)}$$

$$\frac{dU}{dQ} = \frac{5}{7}$$

Q.59 (2)

As Volume decreases

$\therefore$ pressure of the gas in the cylinder increases

Q.60 (1)

AB $\rightarrow$ isothermal

$P_A V_A = P_B V_B$ ...(i)

BC $\rightarrow$ Adiabatic

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$ ...(ii)

CD $\rightarrow$ Isothermal

$P_C V_C = P_D V_D$ ...(iii)

DA $\rightarrow$ Adiabatic

$P_D V_D^{\gamma} = P_A V_A^{\gamma}$ ...(iv)

From (i), (ii), (iii) and (iv)

$$\frac{V_B}{V_C} = \frac{V_A}{V_D}$$

Q.61 (1)

For adiabatic

$T V^{\gamma-1} = C\ (\gamma > 1)$ ....(i)

For isothermal T = const ....(ii)

From (i) and (ii)

$T_2 < T_1$

Q.62 (3)

For isothermal

$PV = C$. or $P_1 \alpha \frac{1}{V_1}$ ...(i)

For adiabatic

$PV^{\gamma} = C$, $P_2 \alpha \frac{1}{V_2^{\gamma}}$ ...(ii)

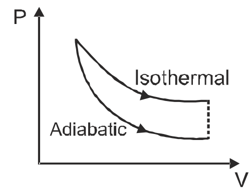
from (i) and (ii)

$P_1 > P_2$

Q.63 (1)

As W.D. by gas in isothermal is more as compare to adiabatic process

$\therefore \Delta W_2 < \Delta W_1$



Q.64 (3)

Isothermal $P \propto \frac{1}{V}$

Adiabatic $P \propto \frac{1}{V^{\gamma}}$

Also, slope of adiabatic is more as compare to isothermal

$\therefore$ option (3) is correct.

Q.65 (3)

Adiabatic process

$\Delta Q = 0$

For any process

$\Delta U = nC_V\Delta T$

Hence, option (3) is correct.

Q.66 (4)

$B = \gamma P$ (for adiabatic process)

$B = 1.4 \times 1 \times 10^5 = 1.4 \times 10^5\ N/m^2$

Q.67 (2)

$B = \frac{VdP}{dV} = -\frac{(-PdV)}{dV}$ (for isothermal process)

$B = P$

Q.68 (2)

Slope $= -\gamma \frac{dP}{dV}$

As slope of A > slope of B

$\therefore \gamma$ of A > γ of B

or A $\rightarrow$ Helium

B $\rightarrow$ Hydrogen

Q.69 (3)

For free expansion

$\Delta U = 0$ or $\Delta T = 0$

$\therefore$ U or T = const

Q.70 (1)

For free expansion

$Q = 0, W = 0, \Delta U = 0$

Q.71 (1)

$\frac{dP}{P} = -\gamma \frac{dV}{V}$ (For adiabatic)

$0.5 = -1.4 \dfrac{dV}{V}$

$\therefore$ Volume decrease by 0.36%

Q.72 (4)
XY Adiabatic compresion
YZ Isothermal Expansion
ZX Compression at constant pressure

Q.73 (1)
Self explainatory

Q.74 (1)
As W.D. is isobaric > W.D. in Isothermal > W.D in adiabatic
or $W_2 > W_1 > W_3$
Hence option (1) is correct.

Q.75 (1)
Process ...(1) is isobaric
$\Delta U_1 = \Delta Q - \Delta W =$ positive
process (2) is isothermal
$\Delta U_2 = 0$
Process (3) is adiabatic
$\Delta Q = 0$
$\Delta U = -\Delta W =$ negative
$\therefore \quad \Delta U_1 > \Delta U_2 > \Delta U_3$

Q.76 (1)
As $\Delta Q = \Delta U + W$
$\Delta U = -W$ (given)
or $\Delta Q = 0$
$\therefore$ Process is adiabatic

Q.77 (4)
For polytropic process $PV^x = k$;

$C = C_V + \dfrac{R}{1-x} \Rightarrow$ As $PV^2 = K$ (given) $\Rightarrow$ Put x = 2

$C = C_V + \dfrac{R}{1-2} = C_V - R.$

$\therefore C < C_V$.

Q.78 (3)
PT = constant

$P\left(\dfrac{PV}{nR}\right) = \text{constant}$

P^2V= constant. Therefore the graph C is suitable.

Q.79 (1)
From the graph shown.

$V_{av} \propto \sqrt{T} \propto \sqrt{PV}$

$V_{av_1} : V_{av_2} : V_{av_3}$

$\sqrt{V_oP_o} : \sqrt{V_o.4P_o} : \sqrt{4V_oP_o}$

1 : 2 : 2

Q.80 (2)
From ideal gas equation
PV = nRT

$PV = \dfrac{m}{M}RT \Rightarrow \dfrac{V}{T} = \dfrac{mR}{MP} = C_B$

In second case

$\dfrac{V}{T} = \dfrac{2mR}{M}$

Q.81 (3)
PV^γ = constant

$V^\gamma \dfrac{dP}{dV} = \gamma\, PV^{\gamma-1}\dfrac{dV}{dV} = 0$

$\dfrac{dP}{dV} = \dfrac{-\gamma\, PV^{\gamma-1}}{V^\gamma} = \dfrac{-\gamma P}{V}$

$= -1.4 \times \dfrac{0.7\times 10^5}{0.0049}$

$= -2\times 10^7$

Q.82 (1)

$$\Delta W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} KVdV \qquad \because \frac{P}{V} = K$$

$$= \frac{KV_2^{\,2} - KV_1^{\,2}}{2} = \frac{P_2V_2 - P_1V_1}{2}$$

$$\Rightarrow \Delta U = \frac{3}{2}R\left(2T_0 - T_0\right) = \frac{3}{2}RT_0$$

$$\Delta Q = \Delta U + \Delta W \;\; = 2RT_0$$

Q.83 (4)

$PV^\gamma = K$

$\ln P + \gamma \ln V = \ln K$

Differentiate both sides

$d(\ln P) + \gamma\, d(\ln V) = 0$

$\dfrac{d(\ln P)}{d(\ln V)} = -\gamma$

$\gamma_B > \gamma_A \Rightarrow$ B is monoatomic

Gas A is diatomic

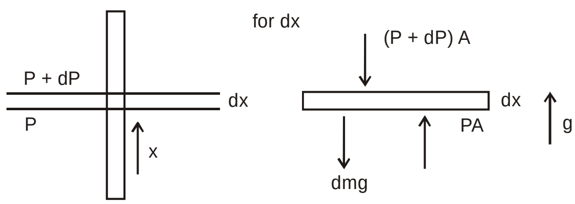
Q.84 (1)

$$T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma}$$

$$T_2 = T_1 \left(\frac{P_1}{P_2}\right)^{1-\gamma/\gamma} = 300\left(\frac{1}{4}\right)^{\frac{1-\frac{4}{3}}{\frac{4}{3}}} = 300\sqrt{2}$$

JEE-ADVANCED OBJECTIVE QUESTIONS

Q.1 (C)



PA – dmg – (P + dP) A = dmg
– AdP = 2dmg
– AdP = 2ρ A dx g

– dP = 2ρ g dx. Where $\rho = \frac{m}{V}$

$$PV = \frac{m}{M}RT$$

$$\frac{PM}{RT} = \frac{m}{V}$$

$$\rho = \frac{PM}{RT}$$

$$-dP = \frac{2PM}{RT}.g.dx$$

$$-\int_{P_o}^{P'_o} dP/P = \frac{2Mg}{RT}\int_{o}^{x=H/2} dx$$

$$\ln\frac{P_0}{P_0'} = \frac{2Mg}{RT}\cdot\frac{H}{2} \Rightarrow \frac{P_0}{P_0'} = e^{MgH/RT}$$

Q.2 (D)
For an ideal gas
$C_P - C_V = R$
If $C_P - C_V = 1.09$ R.
or $p_A > p_B$ $T_A < T_B$
Then gas will be real . Thus pressure is high and temperature is low for real gas.

Q.3 (D)
$C_p = 3.5$ R (At STP)
As temperature increases, vibrational degree of freedom becomes 2 at higher temperature.

$$C_P = \frac{9}{2} R = 4.5 R$$

Q.4 (B)
Average velocity will be same for same temperature.

Q.5 (D)
Work done by gas = Area under P-V diagram

$$= \frac{\pi\,(4-3)(4-2)}{2} + \frac{\pi\,(2-1)(3-2.5)}{2}$$

$$= \frac{2.5\,\pi}{2} = \frac{5\,\pi}{4} \text{ atm L}$$

$W = -\left(\frac{5\,\pi}{4}\right)$ atm L (Work done by gas is negative as cycle is anticlockwise)

Q.6 (D)
$(p_0, v_0) \rightarrow (p_0, 2v_0)$

$$\Delta U_1 = \frac{3}{2}nR\Delta T\,;\ \Delta U_2 = \frac{5}{2}nR\Delta T$$

$\Delta U_2 > \Delta U_1$; $\Delta W_1 = \Delta W_2$
$\Delta Q_1 - \Delta Q_2 = \Delta U_1 - \Delta U_2$
$\Delta U_2 + \Delta W_2 > \Delta U_1 + \Delta W_1$

Q.7 (C)
From the graph shown the equation of line is

$$P - P_0 = \left(\frac{\frac{P_0}{2} - P_0}{2V_0 - V_0}\right)(V - V_0)$$

$$P - P_0 = \frac{-P_0}{2V_0}(V - V_0)$$

$$P = \frac{-P_0 V}{2V_0} + \frac{3P_0}{2}$$

Now we know PV = nRT

$$\Rightarrow \left(\frac{3}{2}P_0 - \frac{P_0 V}{2V_0}\right)V = nRT$$

For maximum temperature

$$\frac{dT}{dV} = 0 \Rightarrow \frac{3}{2}P_0 - \frac{P_0 V}{V_0} = 0$$

$$V = \frac{3}{2}V_0$$

$$T_{max} = \left(\frac{3}{2}P_0 - \frac{P_0}{2V_0}\cdot\frac{3}{2}V_0\right)\frac{3}{2}V_0\cdot\frac{1}{nR}$$

$$= \frac{3}{4}P_0\cdot\frac{3}{2}V_0\cdot\frac{1}{R} = \frac{9P_0V_0}{8R}$$

Q.8 (B)

Initially

$$PV_1 = \frac{12}{M}RT_1$$

or $P\left(4\times10^{-3}\right) = \frac{12}{M}R\left(273+7\right)$ ..... (1)

$$\rho = \frac{m}{V_2} = \frac{12}{V_2} = 6\times10^{-4}\ \text{gm/cc}$$

$$P\left(\frac{12}{6\times10^{-4}}\right)\times\left(10^{-2}\right)^3 = \frac{12}{M}R(T) \text{ ..... (2)}$$

from $1 \div 2$

$$\frac{4\times10^{-3}}{12\times10^{-6}}\times6\times10^{-4} = \frac{273+7}{T} \Rightarrow T = 1400\,K$$

Q.9 (D)

$$\Delta Q = \Delta U + \Delta Q$$

$$2C\Delta T = n\frac{f}{2}R\Delta T + PdV$$

$$2C\Delta T = 2\times\frac{5}{2}R\Delta T + PdV \text{ ____(1)}$$

$$\frac{PT^2}{V} = K$$

or $\frac{T^3}{V^2} = \frac{K}{nR} \Rightarrow T^3 = \frac{K}{nR}V^2$

$$\Rightarrow 3T^2dT = \frac{K}{nR}2VdV$$

or $\frac{3T^2}{2V}dT = \frac{K}{nR}dV$

$$\frac{3}{2}dT = \frac{P}{nR}dV \text{ ____(2)}$$

From (1) and (2)

$$2C\Delta T = 5R\Delta T + nR\frac{3}{2}dT$$

$2C = 5R + 3R$

$2C = 8R$

So, molar heat capacity C = 4R

Q.10 (B)

$$\Delta U = n\frac{f}{2}R\ \ \Delta T$$

For Isobaric process $V_1 \to T_1 = \frac{P_1V_1}{nR}$

At $V_2 \to T_2 = \frac{P_1\left(V_1/2\right)}{nR} = \frac{T_1}{2}$

$$\Rightarrow \Delta U_P = \frac{nfR}{2}\left[\frac{T_1}{2}\right] \text{ ________(1)}$$

Isothermal $\Delta U_T = 0$ ________(2)

Adiabatic $PV^\gamma = K$

$TV^{\gamma-1} = K$

$$\frac{T_1}{T_2} = \frac{V_2^{\gamma-1}}{V_1^{\gamma-1}} = \frac{1}{2^{\gamma-1}}$$

$$T_2 = 2^{\gamma-1}T_1 > \frac{T_1}{2}$$

$$\Delta U_{Adiabatic} = n\frac{F}{2}R\left[2^{\gamma-1}\right]\frac{T_1}{2}$$

$$\Delta U_{adiabatic} = \Delta U_p\left(2^{\gamma-1}\right) \text{ ______(3)}$$

Q.11 (A)

For larger n, pressure will be smaller, so work done will be smaller for larger n.

Q.12 (B)

$V = kT^{2/3}$

$$dV = \frac{2}{3}\ kT^{-\frac{1}{3}}\,dT$$

$$W = \int PdV = \int\frac{nRT}{V}dV$$

$$= R\int\frac{T}{V}dV = R\,\frac{2}{3}\int\frac{T\ KT^{-\frac{1}{3}}dT}{KT^{\frac{2}{3}}} = \frac{2}{3}R\,(T_2 - T_1)$$

$$= \frac{2}{3}R\ (30) = 20\ (8.31) = 166.2\ J$$

Q.13 (C)

$VP^n =$ constant.

$dV\,P^n + VnP^{n-1}\,dP = 0$

$-\dfrac{VdP}{dV} = \dfrac{P}{n}$ = bulk modulus

Q.14 (C)

$V = k\left(\dfrac{nRT}{VT}\right)^{0.33}$

$V^{1.33}$ = const

V = const

∴ proceses is isochoric

Q.15 (A)

Correct graph is shown in option (A)

Process 1–2 adiabatic process, Process 2–3 Isochoric process, process 3–1 Isothermal process.

Q.16 (A)

For adiabatic process

$PV_1^{\gamma} = P_A V_2^{\gamma}$

$P_A = P\left(\dfrac{V_1}{V_2}\right)^{\gamma}$ ......(1)

For isothermal process

$PV_1 = P_B V_2$

$P_B = P\,\dfrac{V_1}{V_2}$ .....(2)

From (1) and (2)

$P_A < P_B$ [For expansion $V_2 > V_1$]

and by PV = nRT

$T_A < T_B$

Q.17 (A)

(P = constant)

$\dfrac{\Delta Q}{\Delta W} = \dfrac{nC_P\Delta T}{nR\Delta T} = \dfrac{C_P}{R} = \dfrac{5}{2}$

Q.18 (D)

Process AB is isobaric [V α T]

$T_B > T_A$

∴ $U_B > U_A$

$W_{BC} < W_{AB}$ (Area under P-V curve)

Q.19 (A)

dW = dQ – dU

$dW = nCdT - nC_VdT$

$W = \int CdT - \int C_VdT$

$= \int \dfrac{a}{T}dT - C_V\,\Delta T$

$= a\ln\left(\dfrac{\eta T_0}{T_0}\right) - \dfrac{(T_2 - T_1)R}{\gamma - 1}$

$W = a\ln\eta\ \dfrac{-(\eta-1)\,T_0\,R}{\gamma-1}$

Q.20 (A)

$T = T_0 + aV^3$

$\Rightarrow \dfrac{PV}{nR} = T_0 + aV^3$

$\Rightarrow P = nR\left[\dfrac{T_0}{V} + aV^2\right]$

For minimum P, $\dfrac{dP}{dV} = O$

$\Rightarrow \dfrac{-T_0}{V^2} + a\,2V = 0 \Rightarrow V = \left(\dfrac{T_0}{2a}\right)^{\frac{1}{3}}$

Q.21 (D)

$P = R\left[\dfrac{T_0}{V} + aV^2\right]$ and $V = \left(\dfrac{T_0}{2a}\right)^{\frac{1}{3}}$

$\Rightarrow P = \dfrac{3}{2}\left(a^{\frac{1}{3}}RT_0^{\frac{2}{3}}\right)2^{\frac{1}{3}}$

Q.22 (D)

$\text{Process}_{1\to2}$ and $\text{Process}_{3\to4}$ are isochoric process.

$W_{12} = 0$

$W_{34} = 0$

$W_{23} = nR(T_3 - T_2)$

= 3 R (2400 – 800) = 4800 R

$W_{41} = nR(T_1 - T_4)$

= 3 R (400 – 1200) = – 2400 R

W = (4800 – 2400) R = 2400 R

= 20 kJ

Q.23 (D)

$\dfrac{V_{sound}}{V_{rms}} = \dfrac{\sqrt{\dfrac{\gamma P}{\rho}}}{\sqrt{\dfrac{3P}{\rho}}} = \sqrt{\dfrac{5}{9}} \Rightarrow \gamma = \dfrac{5}{3}$ [Monoatomic gas]

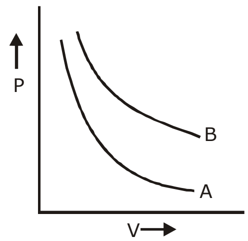
PT = const

$P^2V = \text{const} \Rightarrow PV^{1/2} = \text{const}$

$\Rightarrow x = \dfrac{1}{2} \Rightarrow C = C_V + \dfrac{R}{1-x} = \dfrac{3}{2}R + 2R = \dfrac{7R}{2}$

Q.24 (C)

PV = Const
Slope of B > slope of A
$n_B > n_A$
$m_B > m_A$



Q.25 (D)

$v_f = \eta v_0$
$W_{gas} = RT_0 \ell n\eta$
$W_{atm} = pdv = pv(\eta - 1) = RT_0(\eta - 1)$
At constant temperature $\Delta U = 0$

Q.26 (A)

PV = nRT Along AB $V\downarrow$ $T\downarrow$

Along BC $P\uparrow$ $T\uparrow$

Along CA $\dfrac{p}{(1/v)}$ = const U = const

$$w = \int Pdv = \int \frac{kdv}{v} = k\,\ell n(v_i / v_f) = -ve$$

Q.27 (B)

$$\frac{P}{V} = \frac{nRT}{V^2}$$

so $\dfrac{T_1}{T_2} = \dfrac{kv_1^2}{kv_2^2} = \dfrac{1}{4} \Rightarrow T_2 = 1200\,K$

$\Delta T = 1200 - 300 = 900\,K$
$\Delta U = 2 \times 3/2\,R \times 900 = 2700\,R$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (C,D)
By energy conservation, energy loss by one molecule is equal to gain by other.

Q.2 (A,B)

$\vec{P} = M\vec{V}_{av}$, As $\vec{V}_{av} = 0$ (in equilibrium)

$\therefore\ \vec{P}_{av} = 0$

Q.3 (A,B,C)

Avg. momentum/mol $\propto \sum V_x^{\,2}$

$\sum V_x^{\,2}$ is same at NTP

$(K.E.)_{avg} \propto T$

$(K.E.)/\text{vol.} \propto T$

Q.4 (B,D)

$$v_{rms} = 1.73\sqrt{\frac{KT}{m}}$$

so v_{rms} does not change

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \Rightarrow \frac{n_1}{n_2} = \frac{1}{2}$$

Q.5 (A,B)

$$\Delta Q = \frac{f}{2} nR\Delta T$$

$$f = \frac{2 \times 3 \times 4.2}{1 \times 8.3 \times 1}$$

$$\begin{cases} n = 1 \\ R = 8.3 \end{cases}$$

$\Delta T = 1$

$$= \frac{6 \times 4.2}{8.3} \cong 3$$

The gas must be monoatomic.

Q.6 (A,D)

$$n_1 = \frac{7}{28};\quad n_2 = \frac{11}{44}$$

So $n_1 + n_2 = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

$$m_0 = \frac{18kg}{(1/2)} = 36kg$$

$$C_{V\,mix} = \frac{\frac{1}{4} \times \frac{5}{2}R + \frac{1}{4} \times 3R}{\frac{1}{4} + \frac{1}{4}} = \frac{11}{4}R$$

$$C_{P\,mix} = \frac{11}{4}R + R = \frac{15}{4}R$$

$$r = \frac{C_P}{C_V} = \frac{15}{11} = \frac{45}{33} \cong \frac{47}{35}$$

Q.7 (A,B,C,D)
Specific heat of a substance can be finite, infinite, zero and negative.

Q.8 (A,B)
$C_P > C_V$
and $C_P - C_V = 2$
$\therefore$ option A and B is correct.

Q.9 (A, B, D)

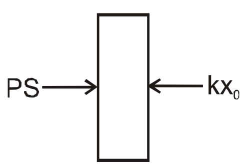
$T = \frac{PV}{nR}$

$\therefore$ initial and final temperature are equal

as $U = \frac{nf\,RT}{2}$

$\therefore$ $U_{initial} = U_{final}$.
Also the net work done by an ideal gas in the process may be zero.

Q.10 (A,D)
For equilibrium of piston



$PS = Kx_0$

$P = \frac{Kx_0}{S}$

For piston
$W_{all} = KE_2 - KE_1$

$W_{gas} - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$W_{gas} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{positive}$

$\Delta Q = 0$
$\Delta Q = \Delta U + W$
$\Delta U = -W = \text{negative}$
As internal energy of gas decreases
$\therefore$ temperature of gas decreases.

Q.11 (A,B,C,D)
For any process $\Delta U = n\,C_V\,\Delta T$,
For Isothermal $\Delta T = 0$
or $U = \text{constant}$
$\Delta Q = 0$ (For adiabatic process)
$\therefore$ $\Delta U + W = 0$
$\Delta U = -W$

Q.12 (A,D)

$$\frac{P^2}{\rho} = C \Rightarrow \frac{P_1^2}{\rho_1} = \frac{P_2^2}{\rho_2}$$

$$\Rightarrow P_2^2 = P^2 \times \frac{1}{2} \Rightarrow P_2 = \frac{P}{\sqrt{2}}$$

$$PV = \frac{m}{W} \times RT = \frac{\rho V}{W}RT$$

$$P = \rho T\frac{R}{W} \Rightarrow P_1 = \rho_1 T_1 R/W$$

$$P_2 = \rho_2 T_2 R/W$$

$$\frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2} \Rightarrow \sqrt{2} = 2\frac{T}{T_2}$$

$$\Rightarrow T_2 = \sqrt{2}T$$

$$\Rightarrow P = \rho\frac{TR}{W} \Rightarrow \frac{P^2}{\rho} = \frac{PTR}{W} = C$$

$$\Rightarrow P \propto \frac{1}{T}$$

Q.13 (C,D)
As the process is carried out suddenly it may be adiabatic and as the conductivity is good enough then may be isothermal.

Q.14 (C,D)
In adiabatic process
$\Delta U \neq 0$
$\Delta T \neq 0$
$PV^{\gamma} = \text{constant}$

Q.15 (B,D)
$\Delta W_A = P_1 \Delta V$
$\Delta W_D = P_2 \Delta V$ $\{P_1 > P_2\}$
$\Delta Q_A = \Delta U_A + \Delta W_A$
$\Delta Q_D = \Delta U_D + \Delta W_D$
$\Delta Q_A - \Delta Q_D = \Delta U_A - \Delta U_D + \Delta W_A - \Delta W_D$
$\Delta Q_A - \Delta Q_D = \Delta W_A - \Delta W_D$
$\{\therefore \Delta U_A = \Delta U_D$
$Q_A > Q_D$

$$W_B = PdV = \int_{v_1}^{v_2} \frac{k}{v}dv = k\ell n\frac{v_2}{v_1}$$

$$W_C = k\ell n\frac{v_2}{v_1}$$

hence $W_B - W_C = 0 \Rightarrow Q_B > Q_C$

$Q_A > Q_B > Q_C > Q_D$

Q.16 (B,D)
in cyclic process.
$\Delta U = 0$
$\Delta Q = \Delta W$ as $\Delta W = +ve$
$\therefore \quad \Delta Q = +ve$
or Net heat energy has been supplied to the system.
in process CA
$\Delta W = 0$ $\quad\quad \Delta U = -ve$ (As T = decreases)
$\therefore$ heat energy is rejected out by system
Teperature at C is maximum

Q.17 (C,D)
$\Delta U = \Delta Q - \Delta W$ is same in both methods as it is a state function

Q.18 (A,C)
in cyclic process $\Delta U_1 + \Delta U_2 = 0$
$\Delta U_{Net} = 0$
$\Delta Q - \Delta W = 0$

Q.19 (A,B)

$$\Delta Q = \Delta U + \Delta W \Rightarrow 25 = \frac{nfR\Delta T}{2} + 0$$

$$25 = \frac{1 \times f \times 25 \times 2}{2 \times 3}$$

f = 3 (monoatomic)

Q.20 (A,B)
$A \to B$ constant pressure
$B \to C$ T = constant
$C \to D$ constant Volume
$D \to A$ T = constant
$\therefore$ clearly, option A and B are constant

Q.21 (A,C)
W = PdV. then W = –ve
As pressure and volume both decreases
$\therefore$ temperature of system decreases

Q.22 (C,D)
From information, the process may be very nearly adibatic. Hence option (C) is correct.

Q.23 (C,D)
$\Delta U = 0$ $\quad\quad$ (Adiabatic)
U = const
nC_vT = const
As O_2 and N_2 are diatomic, so there temp are equal but is different from He
For adiabatic PV^γ = const
For O_2, N_2 value of γ is same
$\therefore$ pressure of O_2, N_2 remains same but different from He

Q.24 (B,C)
Slope of × > slope of y
During expansion
$W_y > W_x$
$U_y > U_x \Rightarrow C_{v_2} > C_{v_1}$
$f_2 > f_1$

Q.25 (A,B,D)

$$\eta = \frac{0/P}{I/P} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}.$$

Q.26 (C)

Heat given : $\Delta Q = n_1 C_{V_1} \Delta T \to$ For gas A [As V = constant $\therefore$ dW = 0]

& for Gas B – $\Delta Q = n_2 C_{V_2} \Delta T$

($\because$ For same heat given, temperature rises by same value for both the gases.)

$\Rightarrow \quad n_1 C_{V_1} = n_2 C_{V_2}$
................(1)

Also, $(\Delta P_B)V = n_2 R\Delta T$ and $(\Delta P_A)V = n_1 R\Delta T$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\Delta P_A}{\Delta P_B} = \frac{2.5}{1.5} = \frac{5}{3}$$

$$\Rightarrow \quad n_1 = \frac{5}{3} n_2$$

Substituting in (1)

$$\frac{5}{3} n_2 \, C_{V_1} = n_2 C_{V_2}$$

$$\Rightarrow \frac{C_{V_2}}{C_{V_1}} = \frac{5}{3} = \frac{(\frac{5}{2}R)}{(\frac{3}{2}R)}$$

Hence, Gas B is diatomic and Gas A is monoatomic.

Q.27 (D)

Since $n_1 = \frac{5}{3} n_2$ Therefore $\frac{125}{M_A} = \frac{5}{3}\left(\frac{60}{M_B}\right)$

(From experiment 1 : W_A = 125 gm & w_B = 60 gm)
$\Rightarrow \quad 5M_B = 4M_A$
The above relation holds for the pair–Gas A : Ar and Gas B : O_2.

Q.28 (B)
No. of molecules in 'A' $= nN_A$

$$= \frac{125}{40} N_A = 3.125 \, N_A$$

(Since $n = \frac{125}{40}$ for Ar)

Q.29 (C)

Internal energy at any temperature T

$= nC_VT$

$= \left(\frac{125}{40}\right)\left(\frac{3R}{2}\right)(300)$

[$\because$ C_V for mono atomic gas $= \frac{3R}{2}$]

$\Rightarrow$ $U_i = 2812.5$ cal.

Q.30 (B)

Let initial temperature and volume be T_0 and V_0. Since the process is adiabatic, the final temperature and volume is $TV^{\gamma-1} = T_0V_0^{\gamma-1}$

($\gamma = \frac{5}{3}$ for mono atomic gas)

$\therefore$ $T = T_0\left(\frac{V_0}{V_0/8}\right)^{\frac{2}{3}} = 4T_0$

$\therefore$ percentage increase in temperature of gas is

$\frac{\Delta T}{T_0} \times 100 = \frac{3T_0}{T_0} \times 100 = 300\%$

Q.31 (C)

Adiabatic Bulk modulus $B = -V\frac{dP}{dV} = \gamma P = \gamma\frac{nRT}{V}$

$\therefore$ $\frac{B_i}{B_f} = \frac{T_0}{V_0} \times \frac{V}{T} = \frac{T_0}{V_0} \times \frac{V_0/8}{4T_0} = \frac{1}{32}$

Q.32 (B)

For adiabatic process $dQ = 0$

$\therefore$ $dU + dW = 0$ or $\frac{dW}{dU} = -1$

Q.33 (A)

In free expansion, temperature of the gas remains constant, therefore

$p_0 v_0 = p.\ 3v_0$

where v_0 = initial volume.

$p = \frac{p_0}{3}$

Q.34 (A)

For adiabatic compression, initial conditions are $\frac{p_0}{3}$ and $3v_0$. Final volume and pressure are v_0 and $3^{2/3}$ p_0.

$\frac{p_0}{3}.(3v_0)^{\gamma} = 3^{2/3}\ p_0(v_0)^{\gamma} \Rightarrow 3^{\gamma-1} = 3^{2/3}$

or $\gamma - 1 = \frac{2}{3} \Rightarrow \gamma = \frac{5}{3}$

i.e. gas is monoatomic

Q.35 (B)

$KE_{avg} \propto T$

Applying $TV^{\gamma-1} = K$ for adiabatic process –

$T_1\ V_1^{\gamma-1} = T_2\ V_2^{\gamma-1}$

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{3v_0}{v_0}\right)^{5/3-1} = 3^{2/3}$

Q.36 (A) p,r,s (B) q (C) p,r,s (D) q,r

(A) If $P = 2V^2$, from an ideal gas equation PV = nRT we get

$2V^3 = nRT$

$\therefore$ with increase in volume

(i) Temperature increases implies dU = +ve

(ii) dW = +ve

Hence dQ = dU + dW = +ve

(B) If PV^2 = constant, from an ideal gas equation PV = nRT we get VT = K (constant)

Hence with increase in volume, temperature decreases

Now $dQ = dU + PdV = nC_vdT - \frac{PV}{T}\ dT$ [$\because$ $dV = -\frac{V}{T}dT$]

$= nC_vdT - \frac{PV}{T}dT = n(C_v - R)\ dT$

$\therefore$ with increase in volume dT = –ve

and since $C_v > R$ for monoatomic gas. Hence dQ = –ve with increases in temperature dV = –ve,

$\therefore$ W = –ve

(C) $dQ = nC\ dT = nC_v\ dT + PdV$

$\Rightarrow$ $n(C_v + 2R)\ dT = nC_vdT + PdV$

$\therefore$ $2nRdT = PdV$ $\therefore$ $\frac{dV}{dT}$ = +ve

Hence with increase in temperature volume increases and vice versa.

$\therefore$ dQ = dU + dW = +ve

(D) $dQ = nC\ dT = nC_v\ dT + PdV$

or $n(C_v - 2R)dT = nC_vdT + PdV$

or $-2nRdT = PdV$

$\therefore$ $\frac{dV}{dT}$ = –ve

$\therefore$ with increase in volume temperature decreases.

Also $dQ = n(C_v - 2R)dT$

For expantion dT = –ve but $C_v < 2R$ for monoatomic gas. Therefore dQ = +ve

with increase in temperature dV = –ve,

$\therefore$ W = –ve

Q.37 (A) p, s (B) s (C) p, s (D) q, r

(A) PV = nRT

$P = (nRT)\frac{1}{V} = (\text{constant})\frac{1}{V}$, $P \propto \frac{1}{V}$

T = constant i.e. isothermal process

As $\frac{1}{V}$ decreases or V increases

$\therefore$ ΔW = positive

and $\Delta Q = \Delta U + \Delta W = \Delta W > 0$ ($\Delta U = 0$)

(B) $\Delta Q = 0$ and V = increases

$\therefore$ ΔW = positive

(C) PV = nRT $V \propto T$ (P = constant)

As volume increases, T also increases

i.e., $\Delta U > 0$

and $\Delta W > 0$ So $\Delta Q > 0$

(D) For cyclic process $\Delta U = 0$

$\Delta W < 0$ (anticlockwise)

$\Delta Q = \Delta U + \Delta W < 0$

NUMERICAL VALUE BASED

Q.1 [435]

Process A $\rightarrow$ B

$$W_{AB} = \int P\,dv = \int \frac{3}{2}T^{1/2}dv$$

$$= \int \frac{3}{2}T^{1/2} \times \frac{1}{3}RT^{-1/2}dT$$

On solving, $W_{AB} = 50\,R = 50 \times 8.3 = 415$ J

Process B $\rightarrow$ C

$$U = \frac{1}{2}V^{1/2}$$

$$\frac{3}{2}RT = \frac{1}{2}V^{1/2}$$

$\Rightarrow$ $3PV^{1/2} = 1$

$\therefore$ $P = \dfrac{1}{3\sqrt{V}}$

Now $W_{BC} = \int P\,dv = \int_{100}^{1600} \frac{1}{3\sqrt{V}}dv = \frac{2}{3}\sqrt{V}$

$= \frac{2}{3}[40 - 10] = \frac{2}{3} \times 30 = 20$ J

Total W = 415 + 20 = 435]

Q.2 [0005]

Temperature is constant

$\Rightarrow$ $\Delta E = 0$, $dW = nRT\dfrac{dv}{v} = nRT\dfrac{Adx}{AL/2}$

$Q = \Delta E + W$

$$\Rightarrow \frac{dW}{dt} = \frac{nRT}{L/2}\frac{dx}{dt}$$

$$\frac{dQ}{dt} = \frac{dW}{dt}$$

$$\Rightarrow k\frac{1}{900}\frac{\Delta T}{L} = \frac{2nRT}{L}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \frac{dx}{dt} = \frac{k \times 27}{900.nRT}$$

$= \dfrac{415.5 \times 27 \times 2}{900 \times 0.5 \times 8.31 \times 300} = \dfrac{1}{200}$ m/s = 5 mm/s

Q.3 [0002]

Q = 7J

$\Delta Q = DU + W$

$7 = nC_v\Delta T + PdV$

$= n\frac{5}{2}R\Delta T + nR\Delta T$

$7 = \frac{7}{2}(nR\Delta T) \Rightarrow nR\Delta T = 2$ J

Q.4 [0075 J]

$\Delta V = \frac{f}{2}nR\Delta T = \frac{5}{2}(P_2V_2 - P_1V_1) = 63$ J

$mgx + \frac{1}{2}kx^2 + P_0Ax = \omega_{gas}$

$\omega_{gas} = 12$ J

$\Rightarrow \Delta Q = 75$ J

Q.5 [2400]

At B and C $(T_B = T_C)$

$$\frac{P_BV_B}{T_B} = \frac{P_CV_C}{T_C}$$

$$\frac{P_BV_A}{T_B} = \frac{P_C 3V_A}{T_C}$$

$P_B = 3P_C$

for line AC $P \propto V$ (straight line through origin)

so $\frac{P_C}{P_A} = \frac{V_C}{V_A} = \frac{3V_A}{V_A}$

$\Rightarrow P_C = 3P_A$

Thus $P_C = 3P_A$; $P_B = 3P_C = 9P_A$ .....(I)

$V_C = 3V_A$; $V_B = V_A$

$T_A = \frac{P_AV_A}{nR}$ .....(II)

from A to B ; sochoric $P\uparrow T\uparrow$

so $T_B > T_A$

for C to A ; both $(P, V)\downarrow$ so $T\downarrow$

Thus from B to C (we could have maximum temperature)

$P = aV + b$

$$\Rightarrow \quad P = -\left(\frac{6P_A}{2V_A}\right) + 12\,P_A$$

$$\Rightarrow \quad P = -\frac{3P_AV}{V_A} + 12\,P_A$$

$PV = nRT$

$$\left(-\frac{3P_AV}{V_A} + 12P_A\right)V = nRT$$

for T_{max} $\frac{dT}{dV} = 0$

$$\frac{-6P_A}{V_A}V + 12P_A = 0$$

$V = 2V_A \Rightarrow P = 6P_A$

$$T = \frac{6P_A(2V_A)}{nR} = \frac{12P_AV_A}{nR}$$

$T_{max} = 12\,T_A = 2400\text{ K}$

Q.6 [3]

$w = m_wgh$

$$Q = \frac{7}{2} \times m_wgh$$

$$= \frac{7}{2} \times 74 \times 9.8 \times 1.2 \approx 3 \times 10^3\text{ J}$$

$\therefore$ n = 3

Q.7 [1]

$$F = \frac{PVg}{RT}(M_{air} - M_{gas})$$

$$\frac{F_{H_2}}{F_{He}} = \frac{M_{air} - M_{H_2}}{M_{air} - M_{He}} = 1.08$$

Q.8 [2]

$$\frac{d\theta}{dt} = \frac{100-0}{R_{eq}}\ ;\ T_B = 40°C,\ T_D = 60°C$$

Q.9 [1]

$$\frac{40-T}{R_H/2} = \frac{T-20}{R_H/2} + \frac{T-0}{R_H/4}$$

T = 15°C

$$\frac{T-0}{R_H/4} = i_H \Rightarrow i_H = 6\text{ J/s}$$

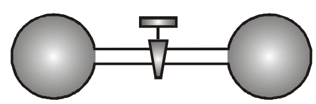
Heat supplied $= 6 \times 5.6 \times 10^4 = 3.36 \times 10^5$ J In 5.6×10^4s.

amount of ice $mL_f = 3.36 \times 10^5$

KVPY

PREVIOUS YEAR'S

Q.1 (B)



After opening of at equilibrium temperature and pressure of whole gas is T_1 and P_1

$$n_1 = \frac{1\times V}{RT},\ n_2 = \frac{0.5\times V\times 4}{RT}$$

$n_1 + n_2 = n$

$$\frac{V}{RT} + \frac{V\times 4}{2RT} = \frac{5VP_1}{RT}$$

$$\frac{3V}{RT} = \frac{5VP_1}{RT_1}$$

$$\frac{P_1}{T_1} = \frac{0.6}{T}$$

$\Delta Q = 0, \qquad \Delta W = 0$

$\therefore\ \Delta U = 0$

$n_1 C_v T + n_2 C_v T = (n_1 + n_2)\, C_v T_1$

$T_1 = T$

$$\frac{P_1}{T} = \frac{0.6}{T}$$

$P_1 = 0.6$ atm

Q.2 (C)

$PV = N \times K \times T$

where K is Boltzmann constant

$10^5 \times 100 = N \times 1.38 \times 10^{-23} \times 273$

$N \approx 3\times10^{27}$

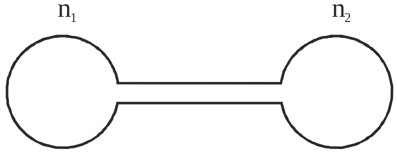
Q.3 (B)

Pressure of gas is app. same everywhere in the vessel

Q.4 (B)

Mole conservation

$n_1 + n_2 = n$



Initial no. of moles $= n_1 = n_2 = \dfrac{n}{2}$

finally when temp of 1 vessel is T & another is 2T

$$n_1 = \frac{PV}{RT}$$

$$n_2 = \frac{PV}{R2T} \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$$

$n_1 + n_2 = n$

$$n_1 = \frac{2n}{3}\ ;\ n_2 = \frac{n}{3}$$

mass of gas $\propto n_1$

$$\therefore\ \frac{M_2}{M_1} = \frac{\frac{n}{3}}{\frac{n}{2}} = \frac{2}{3}$$

Q.5 (A)

$$n = \frac{PV}{RT} = \frac{10^5 \times 1}{\frac{25}{3}\times 300} = 40$$

$N = 40 \times 6.023 \times 10^{23} = 24 \times 10^{24}$

Average sep. $= \left(\dfrac{1}{n}\right)^{1/3} = 1\text{nm}$

Q.6 (A)

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{M_0}}$$

$P = N \times 2mV_{rms}$

$1.01 \times 10^5 = N \times 2 \times 5 \times 10^{-27} \times V_{rms}$

$$N = \frac{1.01\times10^5\sqrt{5\times10^{-27}}}{2\times5\times10^{-27}\sqrt{3\times1.4\times10^{-23}\times293}}$$

$= 6.43 \times 10^{27}$

Q.7 (B)

energy conservation

$$\frac{f_1}{2}n_1RT + \frac{f_2}{2}n_2RT_2 = \frac{f_{eq}}{2}(n_1+n_2)RT$$

$$f_{eq} = \frac{f_1n_1 + f_2n_2}{n_1+n_2}$$

$$\Rightarrow \frac{2}{y_{eq}-1} = \left(\frac{n_1}{n_1+n_2}\right)\left(\frac{2}{y_1-1}\right) + \left(\frac{n_2}{n_1+n_2}\right)\left(\frac{2}{y_2-1}\right)$$

put $n_1 = 1$ and $n_2 = n$

$\Rightarrow \quad n = 2$

Q.8 (D)

$PV = n_1R\,300$ and $(4V)(5P) = n_2R\,400$ .....(1)

$\Rightarrow$ gas will move from high pressure to law pressure means C_2 to C_1

after long time final pressure P_0

$P_0V = n'_1R300$ and $P_0(4V) = n'_2R(400)$ .....(2)

now $n_1 + n_2 = n'_1 + n'_2$

$$\Rightarrow \frac{PV}{R300} + \frac{5PV}{R100} = \frac{P_0V}{R300} + \frac{4P_0V}{R400}$$

$$\Rightarrow \frac{P}{3} + 5P = \frac{P_0}{3} + P_0$$

$$\Rightarrow \frac{16P}{3} = \frac{4P_0}{3}$$

$\Rightarrow \quad P_0 = 4P$

$$\text{Now, } \frac{n'_2}{n'_1} = \frac{\frac{4P_0V}{R400}}{\frac{P_0V}{R300}} = 3$$

Q.9 $210\text{ g} + \rho_{in}Vg = \rho_0 Vg$

ρ_{in} = density of air inside the balloon

ρ_o = density of air outside the balloon

$$\rho_o - \rho_{in} = \frac{210}{V} = \frac{210}{\frac{4}{3}\pi r^3}$$

$$\frac{PM}{R}\left(\frac{1}{T_0} - \frac{1}{T_{in}}\right) = \frac{210 \times 3}{4\pi r^3}$$

$$\frac{1}{T_0} - \frac{1}{T_{in}} = \frac{630 \times 8}{4\pi(11.7)^3} \times \frac{8.31}{10^5 \times 30 \times 10^{-3}} \approx 0.0007$$

$T_{in} \approx 384 = 111°C$

closest answer the option (C)

Q.10 (B)

$PV^{\gamma} = C$

$P^{1-\gamma}T^{\gamma} = C$

$(0.28)^{1-\gamma} \times (233)^{\gamma} = 1^{1-\gamma} \times T^{\gamma}$

$\gamma = \frac{7}{5}$

$(0.28)^{1-7/5} \times (233)^{7/5} = 1^{1-7/5} \times T^{7/5}$

$T^{7/5} = 233^{7/5} \times (0.28)^{-2/5}$

$$T = \frac{233}{(0.28)^{2/7}}$$

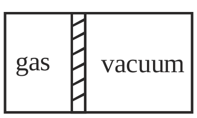
T is coming

more than 298 K or 25°C

$\therefore$ T is more than 25°C

so to cool it an extra ac is required.

Q.11 (A)



expansion is against

vacuum $\therefore \Delta W = 0$

Insulated container $\therefore \Delta Q = 0$

first law of thermodynamics

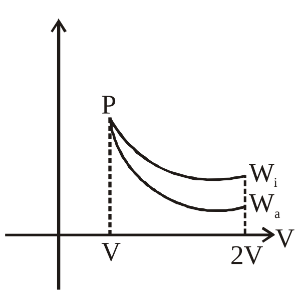
$\Delta Q = \Delta W + \Delta U$

$0 = 0 + \Delta U$

$0 = 0 + \Delta U$

$\Delta U = 0$

Q.12 (C)



$W_i > W_a > 0$

Q.13 (A)

$$\frac{P_1V_1}{n_1} = \frac{P_2V_2}{n_2}$$

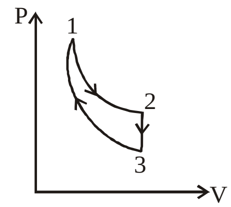
$$\frac{P_1V_1}{n_1} = \frac{P_1\frac{V}{3}}{n_2} \Rightarrow n_2 = \frac{n_1}{3}$$

Now, $\frac{2}{3}$ of Gas will come out to make the presence P_1

Hence 66.66%

Q.14 (D)

$\Delta Q_{1\to2} = \Delta W_{12}$



$W_{total} = \Delta W_{12} + \Delta W_{31}$

$10 = \Delta W_{12} - 20$

$\Delta Q_{12} = \Delta W_{12} = 30$ J

Q.15 (C)

from graph

Q.16 (B)

For adiabatic process

$PV^{\gamma} = C$

$$P\gamma V^{\gamma-1} + \frac{dp}{dv}v^{\gamma} = 0$$

$$\gamma P = -V\frac{dp}{dv}$$

Hence n = 1

Q.17 (A)

Q.18 (B)

$PV^2 = C$

$$\Rightarrow \left(\frac{nRT}{V}\right)V^2 = C$$

$\Rightarrow TV = C$ $\Rightarrow T_1V_1 = T_2V_2$

$\Rightarrow$ If temperature increases, volume decrease and vice versa.

$\Rightarrow V_2 > V_1$ then $T_2 < T_1$

Q.19 (D)

Monoatomic gas $\Rightarrow \gamma = \frac{5}{3}$

n = 1

$PV^3 = C$ on comparing with $PV^\alpha = C$

Here $\alpha = 3$

Heat capacity

$$C = \frac{R}{\gamma-1} - \frac{R}{\alpha-1}$$

$$C = \frac{R}{\left(\frac{2}{3}\right)} - \frac{R}{(2)}$$

$$C = R\left[\frac{3}{2} - \frac{1}{2}\right]$$

$C = R$

Q.20 (B)

Ideal gas equation PV = nRT

For isobaric process

$$V = \left(\frac{nR}{P}\right) T \ (V \propto T(\text{straight line}))$$

$$\text{Slope of line} = \left(\frac{nR}{P}\right)$$

$$\text{slope} \propto \frac{1}{P}$$

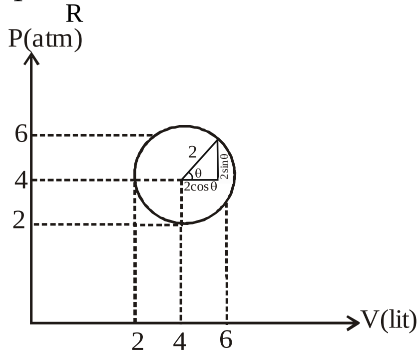
$\text{slope}_3 > \text{slope}_2 > \text{slope}_1$

$P_3 < P_2 < P_1$

Q.21 (A)

[Note : No. of mole of gas is not given, we have assumed no, of mole = 1]

$$T = \frac{PV}{R}$$



T will be maximum when PV is maximum

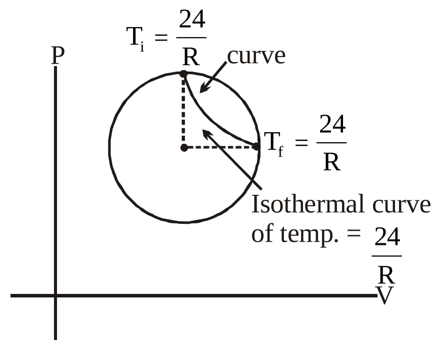
$$T = \frac{PV}{R} = \frac{(4+2\sin\theta)(4+2\cos\theta)}{R}$$

As sinθ and cosθ both

can not be equal to 1 for same value of θ

$\therefore$ T can not be $\frac{36}{R}$

T_{max} should be less than $\frac{36}{R}$



curve is above isothermal curve

$\therefore$ temp. is more than $\frac{24}{R}$ on the given process

So T_{max} lie between $\frac{24}{R}$ and $\frac{36}{R}$

only one option is present

Q.22 (A)

Adiabatic process

$TV^{\gamma-1} = C$

$$\gamma = 1 + \frac{2}{f}$$

$$TV^{\frac{2}{f}} = C$$

$$C_V = \frac{fR}{2} = \frac{3R(1+aRT)}{2}$$

$$\frac{fR}{2} = \frac{3Re^{aRT}}{2}$$

$$\frac{2}{f} = \frac{2}{3e^{aRT}}$$

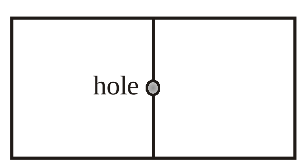
$$TV^{\frac{2}{3eaRT}} = C$$

$$TV^{\frac{3e^{aRT}}{2}} = C$$

Ans. given is $TV^{\frac{3}{2}}e^{aRT}$ So no option is matching may be due to printing mistake.

Q.23 (C)

As dimension of hole is very small than mean path, then at equilibrium effusion rate of gas in both direction must be equal.



For this $\dfrac{P_1}{\sqrt{T_1}} = \dfrac{P_2}{\sqrt{T_2}}$

Mean free path $\propto \dfrac{T}{P}$

$$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} \times \frac{P_2}{P_1}$$

$$\frac{T_1}{T_2} \times \frac{\sqrt{T_2}}{\sqrt{T_1}}$$

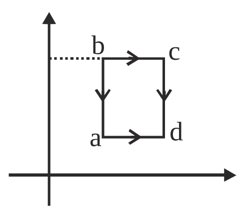
$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{150}{300}} = 0.7$$

Q.24 (A)

In sudden expansion gas do not get enough time for exchange of heat.

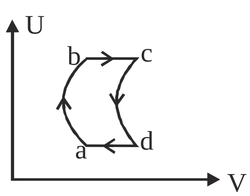
$\therefore$ Process is adiabatic.

Q.25 (A)

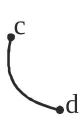


bc $\Rightarrow$ Isothermal process so U remain constant

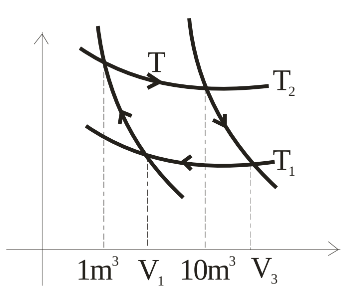
cd $\Rightarrow$ Isentropic process so S remain constant



bc should be straight line parallel to & cd graph should

be


Q.26 (D)



$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{2/3}$$

$$-\frac{1}{4} = \left(\frac{1}{V_1}\right)^{2/3} \Rightarrow V_1 = 8\ m^3$$

Q.27 (A)

$$U = \frac{5}{2}PV + c = \frac{5}{2}nRT + C$$

$f = 5$ $\gamma = 7/15$

$PV^{7/5}$ =constant $\Rightarrow P^5V^7$ = constant

Q.28 (C)

(A), (B) & (D) are wrong and (C) is correct

Q.29 (D)

$\Delta U = f/2\ (\Delta P)\ V = 250$ J

Q.30 (B)

$$W = nRT\ \text{In}\left(\frac{V_2}{V_1}\right) + \frac{nR\Delta T}{1-\gamma}$$

and $TV_2^{r-1} = T_f V_f^{\gamma-1}$

$$V_2 = \left(\frac{T_f}{T}\right)^{\frac{1}{\gamma-1}} V_f$$

V_2 is greater for monotonic

Q.31 (A)

$\Delta W = -$

$\Delta U = 0$

$\Delta Q = -$

Q.32 (A)

$PV^2 = B$ = constant

& $PV = nRT$

$\Rightarrow$ $nRTV = \beta$ = costant

from initial condition of T & V

$\beta = 0.073$ pa – m^6

Q.33 (B)

$$\eta = \frac{W}{Q_{in}}$$

$$\Rightarrow W = \eta Q_{in}$$

$$Q = \int C\ dt$$

For maximum amount of work, efficiency should be maximum, means we have to assume carnot engine.

$\therefore \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{200}{T}$

$\therefore W = \int nQ_{in} = -\int_{600}^{400}\left(1 - \frac{200}{T}\right) CdT$

$= -C\left[T - 200\ell nT\right]_{600}^{400}$

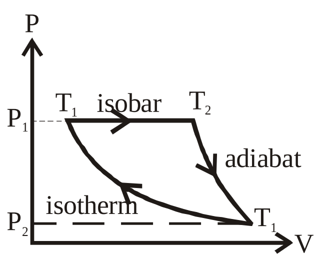
$W = -C\left[-200 + \ell n\left(\frac{3}{2}\right)200\right]$

C = 1 (Given)

$\therefore W = 200 - 200\ell n\left(\frac{3}{2}\right)$

$W = 200\left(1 - \ell n\left(\frac{3}{2}\right)\right)$

Q.34 (B)



$\frac{C_P}{C_P - R} = \gamma$

$C_P = \gamma C_P - \gamma R$

$\gamma R \Rightarrow (\gamma - 1)C_P$

$\frac{\gamma R}{\gamma - 1} = C_P$

$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$

$\eta = \frac{nC_P\Delta T - nRT\ell n\left(\frac{P_1}{P_2}\right)}{nC_P\Delta T} = \frac{1}{2}$

$nC_P\Delta T = 2nRT\ell n\frac{P_1}{P_2}$

$\left(\frac{P_1}{P_2}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma}$

$\frac{\gamma R}{\gamma - 1}(T_2 - T_1) = 2RT_1\ell n\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}}$

$T_2 - T_1 = 2T_1\ell n\left(\frac{T_2}{T_1}\right)$

$x - 1 = \ell n(x^2)$

$x^2 = e^{x-1}$

Option (B)

Q.35 (A)

For ideal gas

PV = nRT

(n = 1), so PV = RT

PV = 8.314 T

Slope of continuous line should be greater than dotted line

JEE-MAIN
PREVIOUS YEAR'S

Q.1 (1)

$dU = nC_V dT = n\frac{5}{2}RdT$

$dQ = nC_P dT = n \times \frac{7}{2}RdT$

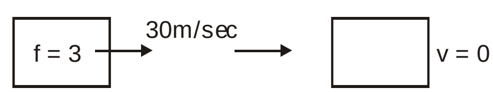
$dW = nRdT = nRdT$

$dU : dQ : dW$

$\Rightarrow n\frac{5}{2}nRdT : n\frac{7}{2}RdT : nRdT$

$5 : 7 : 2$

Q.2 [3600]



ki + ui = kf + uf

$\frac{1}{2}m_{gas}v^2 + \frac{f}{2}nRT_i = 0 + \frac{f}{2}nRT_f$

$\frac{3}{2}nR(T_f - T_i) = \frac{1}{2}m_{gas}v^2$

$\frac{3}{2}(1)R[\Delta T] = \frac{1}{2}(4)(30)^2$

$\Delta T = \frac{1200}{R} = \frac{x}{3R} \Rightarrow x = 3600$

Q.3 (3)

Maxwell's Boltzmann distribution curve is always drawn for no. of molecules (N) vs velocity of molecules. so statement-1 is false.

T.K.E. of diatomic molecule = $\frac{3}{2}KT$

R.K.E. of diatomic molecule = $\frac{2}{2}KT$

Statement-2 is false.

Q.4 [2.55]

using energy conservation

$$\frac{f}{2}\times 2\times 4.5+\frac{f}{2}\times 3\times 5.5=\frac{f}{2}\times P\times 10$$

$$2\times\frac{9}{2}+3\times\frac{11}{2}=P\times 10$$

$$\frac{18}{2}+\frac{33}{2}=P\times 10$$

$$P=\frac{51}{20}$$

P = 2.55 atmosphere

Q.5 (3)

$U = 3PV + 4$

$$\frac{f}{2}PV=3PV+4$$

$$f=6+\frac{8}{PV}$$

$f > 6 \therefore$ Polyatomic gas.

Q.6 [400]

$$V_{rms}=\sqrt{\frac{3RT}{M_0}}$$

$$200=\sqrt{\frac{3R\times 300}{M_0}}$$

$$\frac{x}{\sqrt{3}}=\sqrt{\frac{3R\times 400}{M_0}}$$

$$\frac{200}{\frac{x}{\sqrt{3}}}=\sqrt{\frac{3}{4}}$$

$$\frac{200\sqrt{3}}{x}=\sqrt{\frac{3}{2}}$$

X = 400 m/s

Q.7 (1)

$$\frac{R}{2}=f$$

Q.8 **(3)**

$PV = (n_1 + n_2 + n_3)RT$

$$P\times V=\left[\frac{16}{32}+\frac{28}{28}+\frac{44}{44}\right]RT$$

$$PV=\left[\frac{1}{2}+1+1\right]RT$$

$$P=\frac{5}{2}\frac{RT}{V}$$

Q.9 (4)

$$\lambda=\frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

λ =102 nm

Q.10 (1)

Since each vibrational mode has 2 degrees of freedom hence total vibrational degrees of freedom = 48

f = 3 + 3 + 48 = 54

$\gamma = 1 + \frac{2}{f} = \frac{28}{27} = 1.03$

Q.11 (2)

Let the final temperature of the mixture be T.

Since, there is no loss in energy.

$\Delta U = 0$

$$\Rightarrow \frac{F_1}{2}n_1R\,\Delta T+\frac{F_2}{2}n_1R\,\Delta T=0$$

$$\Rightarrow \frac{F_1}{2}n_1R\,(T_1-T)+\frac{F_2}{2}n_2R\,(T_2-T)=0$$

$$\Rightarrow T=\frac{F_1n_1RT_1+F_2n_2RT_2}{F_1n_1R+F_2n_2R}\Rightarrow\frac{F_1n_1T_1+F_2n_2T_2}{F_1n_1+F_2n_2}$$

Q.12 (2)

(2) f = 4 + 3 + 3 = 10

assuming non linear

$$\beta=\frac{C_p}{C_v}=1+\frac{2}{f}=\frac{12}{10}=1.2$$

Q.13 (1)

Energy associated with each degree of freedom per molecule = $\frac{1}{2}k_BT$.

Q.14 (3)

$$Vrms = \sqrt{\frac{3RT}{M}}$$

$$Vavg = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$$

Q.15 (4)

Q.16 (1)

Q.17 (3)

Q.18 (3)

Q.19 (4)

Q.20 (3)

Q.21 (1)

Q.22 (3)

Q.23 (1)

$$V_{RMS} = \sqrt{\frac{3RT}{M_W}}$$

At the same temperature $V_{RMS} \propto \frac{1}{\sqrt{M_W}}$

$\Rightarrow V_H > V_O > V_C$

Option (1)

Q.24 (3)

$PV = nRT$

$$400 \times 10^3 \times 100 \times 10^{-6} = n\left(\frac{25}{3}\right)(300)$$

$$n = \frac{2}{25}$$

$n = n_1 + n_2$

$$\frac{2}{25} = \frac{M_1}{2} + \frac{M_2}{32}$$

Also $M_1 + M_2 = 0.76$ gm

$$\frac{M_2}{M_1} = \frac{16}{3}$$

Q.25 (1)

Q.26 (3)

Q.27 [500]

Given

Translation K.E. of N_2 = K.E. of electron

$$\frac{3}{2}kT = eV$$

$$\frac{3}{2} \times 1.38 \times 10^{-23} T = 1.6 \times 10^{-19} \times 0.1$$

$\Rightarrow T = 773k$

$T = 773 - 273 = 500°C$

Q.28 [50]

$PV^{-3} = K$

$PV^x = K$

$X = -3$

$$W = -\left[\frac{nR\Delta T}{x-1}\right] = -\left[\frac{nR(200)}{-3-1}\right] = 50\ (nR)$$

Q.29 (2)

$PV^{1/2} = C$

$\therefore\ TV^{-1/2} = C$

$$\therefore\ \frac{T}{\sqrt{V}} = C$$

$$\therefore\ \frac{T_1}{\sqrt{V_1}} = \frac{T_2}{\sqrt{V_2}}$$

$$\therefore\ \left(\frac{T_2}{T_1}\right)^2 = \frac{V_2}{V_1} = 2$$

$$\therefore\ \frac{T_2}{T_1} = \sqrt{2}$$

Q.30 [208 K]

$$\frac{W}{Q_{in}} = \frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4} \qquad .....(i)$$

$$\frac{W}{Q_{in}} = \frac{1}{2} = 1 - \frac{(T_2 - 52)}{T_1}$$

$$\frac{T_2}{2} = T_2 - \frac{3}{4}T_1 + 52$$

$T_1 = 208$ K

Q.31 [60]

$V = KT^{2/3}$

$V^{3/2} = (K)^{3/2}T$

$\therefore$ $TV^{-3/2}$ = const. $\Rightarrow$ x–1 = –3/2

$\therefore$ x = –1/2

$$\therefore \quad \omega = \frac{nR\Delta T}{-x+1}$$

$$= \frac{1(R)(90)}{+\frac{1}{2}+1} = 60R$$

Q.32 (2)

$W_{AB} = 2P_1V_1 \ell n 2$

$W_{BC} = -P_1V_1$

$W_{CA} = 0$

$W_{ABCA} = (2P_1V_1 \ell n 2 - P_1V_1)$

$= nRT(2 \ell n 2 - 1)$

Q.33 (1)

P-iv ; Q-iii ; R-ii ; S-i

Q.34 (1)

$W_{AB} = nRT \ln 2 = RT \ln 2$

$W_{BC} = 0$

$$W_{CA} = \frac{PV - \frac{P}{4} \times 2V}{1-\gamma} = \frac{PV}{2(1-\gamma)}$$

$$W_{ABCA} = RT \ln 2 + \frac{RT}{2(1-\gamma)}$$

$$RT = \left[\ln 2 - \frac{1}{2(\gamma - 1)}\right]$$

Q.35 (1)

Heat and work are treated as path functions in thermodynamics.

$\Delta Q = \Delta U + \Delta W$

Since work done by gas depends on type of process i.e. path and ΔU depends just on initial and final states, so ΔQ i.e. heat, also has to depend on process is path.

Q.36 (113)

$$n = 0.60 = 1 - \frac{T_L}{T_H}$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow TL = 0.4 \times 400$$

= 160 K

= –113°C

Q.37 (4)

$$\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1} \quad (Q\ W = Q_1 - Q_2)$$

$$\frac{400}{800} = 1 - \frac{W}{Q_1}$$

$$\frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

Q_1 = 2W = 2400 J

Q38 (2)

(2) Option (a) is wrong ; since in adiabatic process V $\neq$ constant.

Option (b) is wrong, since in isothermal process

T = constant

Option (c) & (d) matches isothermes & adiabatic formula :

$TV^{\gamma-1}$ = constant & $\frac{T^{\gamma}}{p^{\gamma-1}}$ = constant

Q.39 (1)

Adiabatic process is from C to D

$$WD = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$= \frac{P_DV_D - P_CV_C}{1-\gamma}$$

$$= \frac{200(3)-(100)(4)}{1-1.4}$$

= –500 JAns. (1)

Q.40 (1)

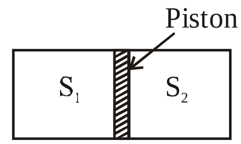
PV^{γ} = constant

Differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

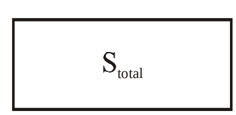
$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

Q.41 (4)



; $S_1 > S_2$

After piston is removed



; $S_{total} = S_1 + S_2$

Q.42 [100]

Q.43 (4)

Q.44 [17258]

Q.45 (2)

Q.46 (4)

Q.47 (2)

Q.48 (2)

Q.49 (4)

Q.50 [500]

Q.51 (1)

Q.52 (1)

Q.53 (1)

Q.54 [480]

$v = 1.5$

$p_1 v_1^{v} = p_2 v_2^{v}$

$(200)\,(1200)^{1.5} = P^2\,(300)^{1.5}$

$P_2 = 200[4]^{3/2} = 1600$ kPa

$$|\text{W.D.}| = \frac{p_2 p_2 - p_1 v_1}{v-1} = \left(\frac{480-240}{0.5}\right) = 480 \text{ J}$$

Q.55 (4)

Q.56 [25]

Pressure is not changing $\Rightarrow$ isobaric process

$$\Rightarrow \Delta U = nC_v \Delta T = \frac{5nR\Delta T}{2}$$

and W = $nR\Delta T$

$$\frac{\Delta U}{W} = \frac{5}{2} = \frac{x}{10} \Rightarrow x = 25.00$$

JEE- ADVANCED
PREVIOUS YEAR'S

Q.1 (A)

Number of moles of He = $\frac{1}{4}$

Now $T_1 (5.6)^{\gamma-1} = T_2 (0.7)^{\gamma-1}$

$$T_1 = T_2 \left(\frac{1}{8}\right)^{2/3}$$

$4T_1 = T_2$

$$\text{Work done} = -\frac{nR[T_2 - T_1]}{\gamma - 1} = -\frac{\frac{1}{4}R\,[3T_1]}{\frac{2}{3}} = -\frac{9}{8}RT_1$$

Q.2 (A) – p,r,t , (B) – p,r (C) – q,s, (D) – r, t

$A \to B \Rightarrow V\downarrow P$ const $\Rightarrow T\downarrow U\downarrow$

(p), (r), (t)

$B \to C \Rightarrow d\omega \downarrow 0$

$P\downarrow T\downarrow$

$d\phi = du + d\omega$

(p), (r)

$C \to D \Rightarrow V\uparrow \Rightarrow T\uparrow$

$du \Rightarrow +ve$

$d\omega = +ve$

(q), (s)

$D \to A \Rightarrow dw \Rightarrow -ve$

(r), (t)

$dq \Rightarrow -ve$

$du = 0$

Q.3 (D)

$$\frac{v_{Rms_{He}}}{v_{Rms_{Ar}}} = \frac{\sqrt{\frac{3RT}{m_{He}}}}{\sqrt{\frac{3RT}{m_{Ar}}}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

Q.4 (D)

$\Delta Q = nC_p \Delta T$

$$= 2\left(\frac{f}{2}R + R\right)\Delta T$$

$$= 2\left[\frac{3}{2}R + R\right] \times 5$$

$$= 2 \times \frac{5}{2} \times 8.31 \times 5 = 208 \text{ J}$$

Q.5 (D)

$$P_1 = \frac{\rho_1 RT}{M_1} \qquad \text{...(i)}$$

$$P_2 = \frac{\rho_2 RT}{M_2} \qquad \text{...(ii)}$$

by (i) and (ii)

$$\frac{\rho_1}{\rho_2} = \frac{8}{9}$$

Q.6 (A,B,C,D)

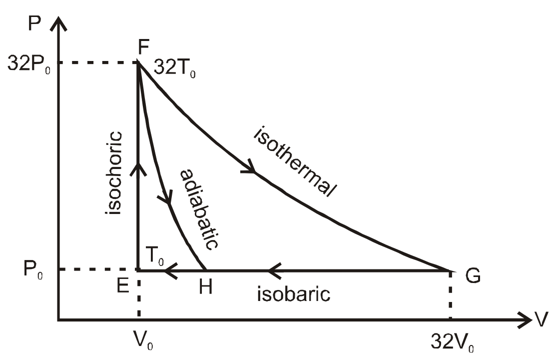
$q = mCT$

$$\frac{dq}{dt} = mc\frac{dT}{dt}$$

R = rate of absorption of heat = $\frac{dq}{dt} \propto C$

(i) in $0 - 100$ k
C increases, so R increases but not linearly
(ii) $\Delta q = mC\Delta T$ as C is more in (400 k – 500 k) then (0 – 100 k) so heat is increasing.
(iii) C remains constant so there no change in R from (400 k – 500 k)
(iv) C is increases so R is increases in range (200 k – 300 k)

Q.7 (A)

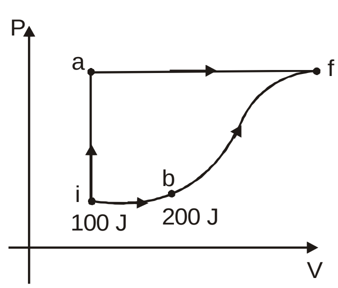


In F→G work done in isothermal proces is nRT ln

$$\left(\frac{V_f}{V_i}\right) = 32\, P_0 V_0 \ln\left(\frac{32V_0}{V_0}\right)$$

$= 32\, P_0V_0 \ln 2^5 = 160\, P_0 V_0 \ln 2$
In G → E, $\Delta W = P_0\, \Delta V = P_0\,(31\, V_0) = 31\, P_0V_0$
In G → H work done is less than $31\, P_0V_0$ i.e., $24\, P_0V_0$
In F → H work done is $36\, P_0V_0$

Q.8 [2]
$w_{ibf} = 150$ J
$w_{iaf} = 200$ J
$Q_{iaf} = 500$ J So $U_{iaf} = 300$ J



So $U_f = 400$ J
$U_{ib} = 100$ J
$Q_{ib} = 100 + 50 = 150$ J
$Q_{ibf} = 300 + 150 = 450$ J

So the required ratio $\frac{Q_{bf}}{Q_{ib}} = \frac{450 - 150}{150} = 2$

Q.9 (D)
Let final temperature of gases is T
Heat rejected by gas in lower compartment ($nC_v\Delta T$) =

$$2 \times \frac{3}{2}R(700 - T)$$

Heat received by gas in above compartment ($nC_p\Delta T$)

$$= 2 \times \frac{7}{2}R(T - 400)$$

Equating above
$2100 - 3T = 7T - 2800$
$\Rightarrow T = 490$ K

Q.10 (D)
$\Delta W_1 + \Delta U_1 = \Delta Q_1$
$\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

$$\frac{7}{2}R\,(T - 400) = \frac{5}{2}R\,(700 - T)$$

$$\Rightarrow \quad T = \frac{6300}{12} = 525 \text{ K}$$

So $\Delta W_1 + \Delta W_2 = 2 \,.\, R.\,(525 - 400) + 2R(525 - 700)$
$= +250R - 350\,R$
$= -100R$

Q.11 (D)

In first process, $\Delta U = W = \frac{P_iV_i - P_fV_f}{\gamma - 1} = 112.5J$

In two-step process, $\Delta Q = \Delta U + (W_1 + W_2)$

$= \Delta U + P_i(V_f - V_i) = 112.5 + 700 = 812.5J$

Hence, (D)

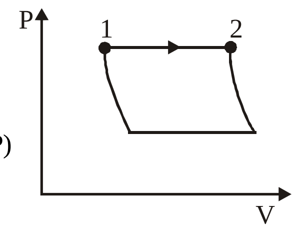
Answer Q.12 Q.13 and Q.14 by appropriately matching the information given in the three columns of the following table.
An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P – V diagrams in column 3 of the table. Consider only path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic process. Here γ is the ratio of heat capacities at constant pressure and constant volume.

The number of moles in the gas is n.
Column-1
Column-2
Column-3

(I) $W_{1\to2} = \frac{1}{\gamma - 1}\left(P_2V_2 - P_1V_1\right)$

(i) Isothermal

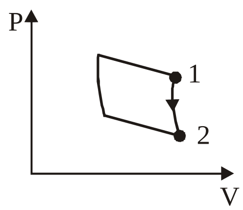
(P)



(II) $W_{1\to2} = -PV_2 + PV_1$

(ii) Isochoric

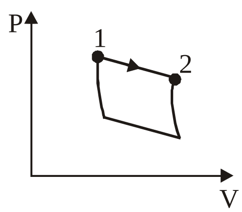
(Q)



(III) $W_{1\to2} = 0$

(iii) Isobaric

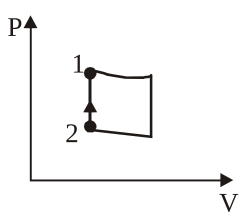
(R)



(IV) $W_{1\to2} = -nRT\ \ln\left(\frac{V_2}{V_1}\right)$

(iv) Adiabatic

(S)



Q.12 (A)

Q.13 (D)

Q.14 (D)
12 to 14

I. $W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

(iv) Adiabatic Q

II. $W = -P\left(V_2 - V_1\right)$

(iii) Isobaric P

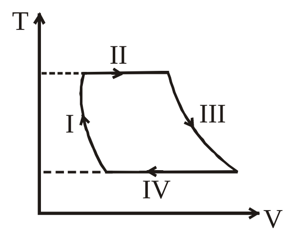
III. $W = 0$

(ii) Isochoric S

IV. $W = -nRT\ln\frac{V_2}{V_1}$

(i) Isothermal R

Q.15 (B, C, D)



(A) Process I is not isochoric, V is decreasing
(B) Process II is isothermal expansion.
$\Delta U = 0, W > 0$
$\Delta Q > 0$
(C) Process IV is isothermal compression.
$\Delta U = 0, W < 0$
$\Delta Q < 0$
(D) Process I and III are NOT isobaric because in isobaric process T $\propto$ V hence isobaric T-V graph will be linear.

Q.16 (900)
$v_i = v$
$v_F = 8v$

For adiabatic process $\left\{\gamma = \frac{5}{3}\right.$ for monoatomic process

$$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$$

$$100(v)^{2/3} = T_2(8v)^{2/3}$$

$$T_2 = 25\ k$$

$$\Delta U = nc_v\Delta T = 1\left(\frac{FR}{2}\right)[100 - 25] = 12 \times 75 = 900\ \text{Joule}$$

Q.17 (C)
Process - I is an adiabatic process
$\Delta Q = \Delta U + W$
$\Delta Q = 0$
$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow$ W < 0
$\Delta U > 0$
$\Rightarrow$Temperatuer of gas increases.
$\Rightarrow$No heat is exchanged between the gas and surrounding
Process - II is an isobaric process
(Pressure remain constant)
$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$
Process - III is an isochoric proces
(Volume remain constant)
$\Delta Q = \Delta U = W$
$W = 0$
$\Delta Q = \Delta U$
Process - IV is an isothermal process
(Temperature remains constant)
$\Delta Q = \Delta U + W$
$\Delta U = 0$

Q.18 (A, B)
From graph

Process $1 \to 2$ is isobaric with $P = \dfrac{RT_0}{V_0}$

Process $2 \to 3$ is isochoric with $V = 2V_0$

Process $3 \to 4$ is isobaric with $P = \dfrac{RT_0}{2V_0}$

Process $4 \to 1$ is isochoric with $V = V_0$

$$\text{Work in cycle} = \frac{RT_0}{V_0}.V_0 - \frac{RT_0}{2V_0}.V_0 = \frac{RT_0}{2}$$

$$Q_{1-2} = nC_p\Delta T = n.\frac{5R}{2}.T_0$$

$$Q_{2-3} = nC_V\Delta T = n.\frac{3R}{2}.T_0$$

$$\therefore \left|\frac{Q_{1-2}}{Q_{2-3}}\right| = \frac{5}{3}$$

$$Q_{3-4} = nC_p\Delta T = n.\frac{5R}{2}.\frac{T_0}{2}$$

$$\therefore \left|\frac{Q_{1-2}}{Q_{3-4}}\right| = 2$$

Q.19 (A,C,D)

$n_1 = 5$ moles $C_{V_1} = \dfrac{3R}{2}$ $\quad P_0V_0T_0$

$n_2 = 1$ mole $C_{V_2} = \dfrac{5R}{2}$

$$(C_v)_m = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{5\times\frac{3R}{2} + 1\times\frac{5R}{2}}{6} = \frac{5R}{3}$$

$$\gamma_m = \frac{(c_p)_m}{(c_v)_m} = \frac{8}{5}$$

$\therefore$ Option 4 is correct

$$(C_p)_m = \frac{5R}{3} + R = \frac{8R}{3}$$

(1) $P_0V_0^{\gamma} = P\left(\dfrac{V_0}{4}\right)^{\gamma}$ Þ $P = P_0(4)^{8/5} = 9.2\,P_0$ which is between $9P_0$ and $10P_0$

(2) Average K.E. $= 5 \times \dfrac{3}{2}RT + 1 \times \dfrac{5RT}{2}$
$= 10RT$

To calculate T

$$\frac{P_0V_0}{T_0} = 9.2P_0 \times \frac{V_0}{4 \times T}$$

so $T = \dfrac{9.2}{4}T_0$

Now average KE $= 10R \times 9.2\dfrac{T_0}{4} = 23RT_0$

(3) $W = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1} = \dfrac{P_0V_0 - 9.2P_0 \times \frac{V_0}{4}}{3/5} = -13RT_0$

Q.20 (3)
(I) Degree of freedom f = 3
Work done in any process = Area under P–V graph
$\Rightarrow$ Work done in $1 \to 2 \to 3 = P_0V_0$

$= \dfrac{RT_0}{3} \Rightarrow (Q)$

(II) Change in internal energy $1 \to 2 \to 3 =$
$\Delta U = nC_v\,\Delta T$

$= \dfrac{f}{2}\,nR\Delta T$

$= \dfrac{f}{2}(P_fV_f - P_iV_i)$

$= \dfrac{3}{2}\left(\dfrac{3P_0}{2}2V_0 - P_0V_0\right)$

$= 3P_0V_0$
$= \Delta U = RT_0 \Rightarrow (R)$

(III) Heat absorbed in $1 \to 2 \to 3$
for any process, 1st law of thermodynamics
$\Delta Q = \Delta W + \omega$

$\Delta Q = RT_0 + \dfrac{RT_0}{3}$

$\Delta Q = \dfrac{4RT_0}{3} \Rightarrow (S)$

(IV) Heat absorbed in process $1 \to 2$
$\Delta Q = \Delta U + W$

$$= \frac{f}{2}(P_f V_f - P_0 V_0) + W$$

$$= \frac{3}{2}(P_0 2V_0 - P_0 V_0) + P_0 V_0$$

$$= \frac{5}{2} P_0 V_0$$

$$= \frac{5}{2}\left(\frac{RT_0}{3}\right)$$

$$\Delta Q = \frac{5RT}{6} \Rightarrow (U)$$

Q.21 (D)

Process $1 \rightarrow 2$ is isothermal (temperature constant)

Process $2 \rightarrow 3$ is isochoric (volume constant)

(I) Work done in $1 \rightarrow 2 \rightarrow 3$

$W = W_{1\rightarrow 2} + W_{2\rightarrow 3}$

$$= nRT \, \text{In}\left(\frac{V_f}{V_i}\right) + W_{2\rightarrow 3}$$

$$= \frac{RT_0}{3} \text{In}\left(\frac{2V_0}{V_0}\right) + 0$$

$$W = \frac{RT_0}{3} \text{In} 2 \Rightarrow (P)$$

(II) ΔU in $1 \rightarrow 2 \rightarrow 3$

$$\Delta U = \frac{f}{2} nR(T_f - T_i)$$

$$= \frac{3}{2} R\left(T_0 - \frac{T_0}{3}\right)$$

$$= \frac{3}{2} R\left(\frac{2T_0}{3}\right)$$

$\Delta U = RT_0 \Rightarrow (R)$

(III) For any system, first law of thermodynamics for $1 \rightarrow 2 \rightarrow 3$

$\Delta Q = \Delta U + W$

$$\Delta Q = RT_0 + \frac{RT_0}{3} \text{In } 2$$

$$\Delta Q = \frac{RT_0}{3}(3 + \text{In } 2) \Rightarrow (T)$$

(IV) For process $1 \rightarrow 2$ (isothermal)

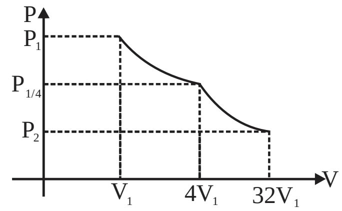
$\Delta Q = \Delta U + W$

$$= \frac{f}{2} nR(T_f - T_i) + nRT \, \text{In}(V_f/V_i)$$

$$= 0 + R\left(\frac{T_0}{3}\right) \text{In}\left(\frac{2v_0}{v_0}\right)$$

$$\Delta Q = \frac{RT_0}{3} \text{ In } 2 \Rightarrow (P)$$

Q.22 (1.77 to 1.78)



$$\frac{P_1}{4}(4V_1)^{5/3} = P_2(32v_1)^{5/3}$$

$$P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$$

$$W_{adi} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{3} - 1}$$

$$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8} P_1V_1$$

$$W_{iso} = P_1V_1 \ln\left(\frac{4V_1}{V_1}\right) = 2P_1V_1 \ln 2$$

$$\frac{W_{iso}}{W_{adio}} = \frac{2P_1V_1 \ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9} \ln 2 = f \ln 2$$

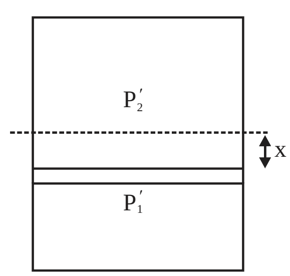
$$f = \frac{16}{9} = 1.7778 \approx 1.78$$

Q.23 (6)

Assuming temperature remains constant at 300 K

From $P_1V_1 = P_2V_2$

$$\frac{P_1\left(\frac{V_0}{2}\right)}{T} = \frac{P_1'\left(\frac{V_0}{2} - Ax\right)}{T}$$



$$(p_1' - P_2')A = mg$$

$$\left[\frac{P_1\left(\frac{V_0}{2}\right)}{\frac{V_0}{2}-Ax}-\frac{P_2\left(\frac{V_0}{2}\right)}{\frac{V_0}{2}+Ax}\right]A = mg$$

$$nRT\left[\frac{1}{4-x}-\frac{1}{4+x}\right]=mg$$

$$(0.1)(8.3)\left[\frac{4+x-4+x}{16-x^2}\right]=mg$$

$$3\left(\frac{2x}{16-x^2}\right)=1$$

$6x=16-x^2$

$x^2+6x-16=0$

$x=2$

distance $=4+2=6$m

Q.24 (2.05)

$W=(\Delta P)_{avg}\times 4\pi R^2 a$

$$=\left|\frac{dP}{2}.4\pi R^2 a\right|$$

{for small change $(\Delta P)_{avg}$ <P> arithmetic mean}

$$=PV^{\gamma}=c \Rightarrow dP=-\gamma\frac{P}{V}dV=-\frac{\gamma P_0}{V}4\pi R^2 a$$

$$=\frac{\gamma P_0}{2V}\times 4\pi R^2 a\times 4\pi R^2 a$$

$$=\frac{\gamma P_0}{2\times 4\pi R^3}4\pi R^2 a\times 4\pi R^2 a$$

$$=\left(4pRP\times a^2\right)\frac{3\gamma}{2}$$

$\therefore x \simeq 2.05$

Q.25 (C)

Process 1

P = constant, Volume increases and temperature also increases

$\Rightarrow$ W = positive , ΔU = positive

$\Rightarrow$ Heat is positive and supplied to gas

Process 2

V = constant, Pressure decrease

$\Rightarrow$ Temperature decreases

$$W=\int pdV=0$$

ΔT is negative and $\Delta U=\frac{f}{2}nR\Delta T$

$\Rightarrow \Delta U$ in negative

$\Delta Q=\Delta U+W$

$\therefore \Delta Q \rightarrow$ Heat is negative and rejected by gas

Process 3

P = constant, Volume decreases

$\Rightarrow$ Temperature also decreases

$W = P\Delta V$ = negative

$$\Delta U=\frac{f}{2}nR\Delta T = \text{negative}$$

$\Delta Q = W + \Delta U$ = negative

Heat is negative and rejected by gas.

Process 4

V = constant, Pressure increases

$$W=\int pdV=0$$

$PV = nRT \Rightarrow$ Temperature increase

$$\Delta U=\frac{f}{2}nR\Delta T \text{ is positive}$$

$\Delta Q = W + \Delta U$ = positive

Ans (C) step 1 and step 4

Q.26 (A)

Q.27 (B)

Elasticity and Thermal Expansion

EXERCISES-I

ELEMENTRY

Q.1 (3)

$$l = \frac{FL}{YA} \Rightarrow l \propto \frac{1}{A}$$

Q.2 (3)

$$l = \frac{FL}{AY} \Rightarrow l \propto \frac{L}{d^2} \Rightarrow \frac{l_1}{l_2} = \frac{L_1}{L_2} \times \left(\frac{d_2}{d_1}\right)^2 = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

Q.3 (1)
Because due to increase in temperature intermolecular forces decreases.

Q.4 (3)
Breaking Force $\propto$ Area of cross section of wire (πr^2)
If radius of wire is double then breaking force will become four times.

Q.5 (1)
In the region OA, stress $\propto$ strain i.e. Hooke's law hold good.

Q.6 (4)
As stress is shown on *x*-axis and strain on *y*-axis

So we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

So elasticity of wire P is minimum and of wire R is maximum

Q.7 (2)

Q.8 (3)

Q.9 (2)

Angle of shear $\phi = \frac{r\theta}{L} = \frac{4 \times 10^{-1}}{100} \times 30° = 0.12°$

Q.10 (3)
Adiabatic elasticity $K\alpha = \gamma P$

Q.11 (1)
Area of hysterisis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heating.

Q.12 (4)

$$U = \frac{1}{2}\left(\frac{YA}{L}\right) l^2\, l. \qquad \therefore\ U \propto l^2$$

$$\frac{U_2}{U_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{10}{2}\right)^2 = 25 \Rightarrow U_2 = 25\,U_1$$

i.e. potential energy of the spring will be 25 *V*

Q.13 (1)

Energy per unit volume $= \frac{1}{2} \times Y \times (\text{strain})^2$

$$\therefore \text{strain} = \sqrt{\frac{2E}{Y}}$$

Q.14 (3)

Q.15 (4)
Increase in tension of wire = $YA\,\alpha\Delta\theta$
$= 8 \times 10^{-6} \times 2.2 \times 10^{11} \times 10^{-2} \times 10^{-4} \times 5 = 8.8$ N

Q.16 (3)
$F = YA\,\alpha\Delta t = 2 \times 10^{11} \times 3 \times 10^{-6} \times 10^{-5} \times (20 - 10) = 60$ N

JEE-MAIN
OBJECTIVE QUESTIONS

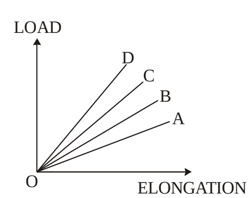
Q.1 (1)
d = 4mm
$Y = 9 \times 10^{10}$ N/m^2

$$\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$$

$$F = AY\frac{\Delta\ell}{\ell} = \pi(2x10^{-3})^2 \, x\, 9 \times 10^9 \, x \, \frac{1}{100} = \pi \, x\, 4 \, x\, 10^{-6} \, x\, 9 \times 10^7 = 360\,\pi\text{ N}$$

Q.2 (3)

$$\frac{F/A}{\Delta\ell/\ell} = Y$$



$$\frac{F}{\Delta\ell} = \frac{Y\pi r^2}{\ell}$$

$$\Rightarrow \frac{F\ell}{Y\pi} \times \frac{1}{\Delta\ell} = r^2$$

$\Rightarrow$ Y & ℓ are same for all then

For same load $r \,\alpha\, \frac{1}{\sqrt{\Delta\ell}}$

Q.3 (3)

$$F = \eta A \frac{x}{h} = 0.4 \times 10^{11} \times 1 \times .005 \times \frac{.02 \times 10^{-2}}{1}$$

$= 4 \times 10^4$ N

Q.4 (3)

$$\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{1 \times 10^5}{1.25 \times 10^{11}} = 8 \times 10^{-7}$$

Q.5 (3)

On heating volume of substance increases while mass of the substance remains the same. Hence the density will decrease

Q.6 (4)

$$K = \frac{AY}{\ell}, K' = \frac{4AY}{\ell/2} = 8K$$

$$\frac{U}{2} = \frac{\frac{1}{2} \times 8K \times \Delta\ell^2}{\frac{1}{2} \times K \times \Delta\ell^2} \Rightarrow U = 16 \text{ J}$$

Q.7 (4)

$$U = \frac{1}{2}\left(\frac{YA}{L}\right) l^2\, l. \qquad \therefore\ U \propto l^2$$

$$\frac{U_2}{U_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{10}{2}\right)^2 = 25 \Rightarrow U_2 = 25\,U_1$$

i.e. potential energy of the spring will be 25 *V*

Q.8 (3)

Given L = 1 mm, $\Delta L = 6 \times 10^{-5}$ mm

$\alpha = 12 \times 10^{-6}\ k^{-1}$

then

$\Delta L = L\alpha\, \Delta T$

6×10^{-5} mm = (1mm) $(12 \times 10^{-6})\, \Delta T$

$\Delta T = 5°C$

Q.9 (1)

Given

L = 25 cm, $A = 0.8 \times 10^{-4}$ cm^2

$\Delta T = 10°C$, $\alpha = 10^{-5}\ °C^{-1}$, $Y = 2 \times 10^{10}\ N^2$

then

$$\frac{\Delta L}{L} = \alpha\Delta T = \frac{F}{AY}$$

$F = \alpha AY\Delta t$

$= (10^{-5})(0{\cdot}8 \times 10^{-4}) \times (2 \times 10^{10}) \times 10$

= 160 N

Q.10 (3)

$L_1 = L + L\alpha_1\, \Delta t$

$L_2 = L + L\alpha_2\, \Delta t$

$$\frac{\text{Stress}_1}{\text{Stress}_2} = \frac{Y_1 L\alpha_1 \Delta t}{L} \cdot \frac{L}{Y_2 L\alpha_2 \Delta t}$$

$$1 = \frac{2}{3}\frac{Y_1}{Y_2} \Rightarrow \frac{Y_1}{Y_2} = \frac{3}{2}$$

Q.11 (3)

$I = CMR^2$

$dI = 2CMRdR = 2CMR\,[R\alpha\Delta T] = 2\alpha I\Delta T$

Q.12 (2)

$$F = AY\frac{\Delta L}{L} = AY\alpha\Delta T$$

$$f = K\sqrt{\frac{F}{\mu}} = K\sqrt{\frac{AY\alpha\Delta T}{\rho A}}$$

$$\Rightarrow f\ \alpha\ \sqrt{\frac{Y\alpha}{\rho}}$$

Q.13 **(2)**

We know that

$U = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume $= \frac{1}{2} \times Y\ (\text{strain})^2$ volume

$$U = \frac{1}{2}\ Y\, (\alpha\Delta T)^2\, AL$$

$U \propto \Delta T^2$

$U \propto (t - 20)^2$

Q.14 (2)

$$\frac{\Delta L}{L} = \alpha\Delta t = -\alpha 20$$

means read more so actual is less

Q.15 (2)

Given L = 20 cm, ΔL_1=0.075 cm, ΔL_2 = 0.045 cm

$\Delta L = L\ \alpha\ \Delta T$

$0{\cdot}075 = 20\ \alpha_1\, (100)$

$0{\cdot}045 = 20\ \alpha_2\, (100)$

Let for third rod L_1 and $L_2 = 20 - L_1$

So $\Delta L_3 = \Delta L_1 + \Delta L_2$

$\Rightarrow 0\cdot06 = L_1\alpha_1\, 100 + (20 - L_1)\,\alpha_2\, 100$

$L_1 = 10$ cm.

Q.16 (1)
Given
f = coefficient of cubical expansion
$\rho_{Spere} = \rho_l'$

$$\Rightarrow \frac{266.5}{\frac{4}{3}\pi\left(\frac{7}{2}\right)^3} = \frac{1.527}{1+35f}$$

$\Rightarrow$ f= 8.3×10^{-4}/.c

Q.17 **(2)**

$$F = A\gamma\frac{\Delta L}{L(1+\alpha\Delta t)}$$

$$F = \frac{A\varepsilon\alpha t}{(1+\alpha t)}$$

Q.18 **(1)**
At 0°C
then $\rho_l v_s g = W_0$ ...(1)
At t°C $\rho_l' v_s' g = W$ ...(2)
$\rho_s v_s = \rho_s' v_s' = m$ ...(3)
$(2) - (1) \Rightarrow (\rho_l' V_s' - \rho_l V_s)\, g = W - W_0$
$W = W_0 + (\rho_l \{1 - \gamma_l t\}\, v_s\, (1 + \gamma_s t) - \rho_l v_s)g$
$= W_0[1 - (\gamma_l - \gamma_s)t]$

Q.19 (1)
$V_\ell > V_e$
$\gamma > 3\alpha$.

Q.20 (3)
$\ell_1(1 + \alpha_1\Delta T) + \ell_2(1 + \alpha_2\,\Delta T) = \ell_f$
$\ell_f = \ell_1 + \ell_2 + (\ell_1\alpha_1 + \ell_2\alpha_2)\Delta T$

$$\ell_f = (\ell_1 + \ell_2)\left(1+\frac{\ell_1\alpha_1 + \ell_2\alpha_2}{\ell_1 + \ell_2}\,\Delta T\right).$$

Q.21 **(3)**
$\alpha_x + \alpha_y$ for x – y plane
$\beta_{CDEH} = 3 \times 10^{-5}$ per °C

Q.22 (4)
$\gamma_{oil} = \gamma_{vessel} \Rightarrow$ D.
Volume increases but mass remains same.

Q.23 (3)
$\because\ \gamma_m < \gamma_{Al}$ $\qquad \rho_m >> \rho_{ac}$
$\Delta V_m < \Delta V_{al}$ $\qquad$ So completely Immersed
$\Delta\rho_m < \Delta\rho_{Al}$ $\qquad$ So $W_2 > W_1$ [$\because$ Displaced mass of alchol is less]

Q.24 (3)

$P\Delta V = nR\Delta T \qquad \Delta V = \frac{nR}{P}\Delta T$

$$\Delta V = \frac{V}{T}\Delta T$$

So , $\gamma = \frac{1}{T}$

Q.25 **(4)**
Initially ρ_s ρ_l and V density of sphere, density of liquid and volume.

$$\frac{B_T' - B_T}{B_T}\times 100 = \frac{V_s'\rho_l'g - V_s\rho_l g}{V_s\rho_l g}\times 100 \Rightarrow [(1 + \gamma_s\,\Delta t)(1 - \gamma_l\,\Delta t) - 1] \times 100$$
$= -\,0.05$ (decreases)

Q.26 (2)

$$\frac{\Delta L}{L}\times 100 = 1 = 100\,\alpha\Delta t = 100\,\alpha(T_2 - T_1)$$

$$\frac{\Delta A}{A}\times 100 = 200\,\alpha\Delta t = 2\%$$

Q.27 (3)
$\Delta L = \Delta L_1 + \Delta L_2$
$(3L)\alpha_{net}\,\Delta t = L\alpha\Delta t = (2L)\,(2\alpha)\,\Delta t$

$$\alpha_{net} = \frac{\alpha + 4\alpha}{3} = \frac{5\alpha}{3}$$

Q.28 **(3)**
V_m denote volume of murcury
$V_{air} = V_{flask} - V_m = V'_{flask} - V_m'$
$V_{flask} - 300 = V_{flask}\,[1 + 3 \times (9 \times 10^{-6})\,\Delta t] - 300\,[1 + 8 \times 10^{-4}\,\Delta t]$

$$V_{flask} = \frac{(300\times 1.8\times 10^{-4})}{27\times 10^{-6}\,\Delta t}\Delta t = 2000\ cm^3$$

Q.29 **(3)**

$$\left.\begin{matrix}\text{Given} & \gamma_l - \gamma_c = c\\ \text{and} & \gamma_l - \gamma_c = s\end{matrix}\right] \Rightarrow\ \gamma_s = c + \gamma_c - s = 3\alpha_s$$

$$\alpha_s = \frac{c + \gamma_c - s}{3}$$

Q.30 **(3)**
$\rho_{0°C}\, h_1\, g = \rho_{30°C}\, h_2\, g$
$\rho_0\ (120) = \rho_0\ (1 - \gamma 30)\ (124)$

$$\gamma = \left(1 - \frac{120}{124}\right)\frac{1}{30} = 11\times 10^{-4}\,/°C$$

Q.31 (1)
On heating the expansion will take place hence both the distances will increase.

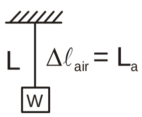
Q.32 **(4)**
at 0°C
$V_{0x} = 20A;$ $V_{0y} = 30A$
Now at time T y read 120°C
So. $V'_{0y} = A(120) = 30A\,(1 + \gamma_m T)$
and $V'_{0x} = Ah = 20A\,(1 + \gamma_m T)$

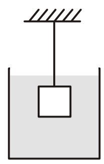
Dividing $\frac{120}{h} = \frac{30}{20}$

$\Rightarrow h = 80.$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (A)





$\Delta\ell_{water} = \ell_w$

$L_a = \frac{WL}{YA}$ $L_w = \frac{\left[W - \frac{W}{\rho_o}\rho_w\right]L}{YA}$

$= \frac{W[1-\frac{\rho_w}{\rho_o}]L}{YA}$

$\frac{L_a}{L_w} = \left[1-\frac{\rho_w}{\rho_o}\right] \Rightarrow \frac{\rho_o}{\rho_w} = \frac{L_a}{L_a - L_w}$

Q.2 (A)

$\frac{F}{A} = \eta\frac{x}{h}$

$\frac{500}{4\times16\times10^{-4}} = 2\times10^6\,\frac{x}{4\times10^{-2}}$

$\Rightarrow$ $x = \frac{5\times10^{-2}}{32}m = 0.156\,cm$

Q.3 (D)
depth = 200 m

$\frac{\Delta V}{V} = \frac{0.1}{100} = 10^{-3}$

density = 1 x 10^3
g = 10

$B = \frac{\Delta p}{\Delta v/v} = \frac{hg\rho}{\Delta v/v}$

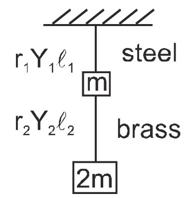
$\Rightarrow$ $B = 200 \times 10 \times 10^3 \times 1000 = 2 \times 10^9$

Q.4 (B)

$\frac{r_1}{r_2} = b$

$\frac{\ell_1}{\ell_2} = a$

$\frac{Y_1}{Y_2} = c$



$\Delta\ell_1 = \frac{(3mg)\ell_1}{A_1Y_1}$

$\Delta\ell_2 = \frac{(2mg)\ell_2}{A_2Y_2}$

$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{3\ell_1}{2\ell_2\,A_1Y_1}\times A_2Y_2 = \frac{3}{2}\,\frac{a}{b^2c} = \frac{3a}{2b^2c}$

Q.5 (C)

$\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$

If Y & $\frac{\Delta\ell}{\ell}$ are constant.

$F = AY\frac{\Delta\ell}{\ell}$

$\Rightarrow$ $F \propto A; \Rightarrow F' = 4F$

Q.6 (D)
$\ell_B = 2m$ $\ell_S = L$
$A_B = 2\ cm^2$ $A_S = 1\ cm^2$

$\Delta\ell_B = \Delta\ell_S$

$\frac{F}{A_B}\frac{\ell_B}{Y_B} = \frac{F}{A_S}\frac{\ell_S}{Y_S}$

$L = \frac{A_SY_S}{A_BY_B}\ell_B = \frac{1}{2}\times\frac{2\times10^{11}}{1\times10^{11}}\times 2 = 2$

Q.7 (D)

$\frac{r_1}{r_2} = \frac{1}{2}$

PE (per unit volume) = $\dfrac{1}{2Y}\left(\dfrac{F}{A}\right)^2$

PE $\propto 1/A^2$

$\dfrac{PE_1}{PE_2} = \dfrac{A_2^2}{A_1^2} = 16 : 1$

Q.8 (B)

Given volume at 0°C = V_0,
cofficient of Linear expansion = a_g
cofficient of cubical expansion = γ_m

$$h = \frac{V_m{}' - V_b{}'}{A_0{}'} = \frac{V_0\left(1+\gamma\Delta T\right) - V_0(1+3a_g\Delta T)}{A_0\left(1+2a_g\,\Delta T\right)}$$

$$= \frac{V_0 T\left(\gamma - 3a_g\right)}{A_0\left(1+2a_g T\right)}$$

Q.9 (C)

Initially $P = \dfrac{V_b\rho_b}{A_c},\ P' = \dfrac{V_b{}'\,\rho_b{}'}{A_c{}'}$

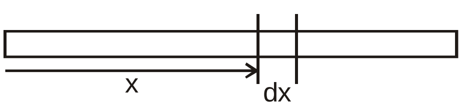
$$P' = \frac{V_b\left(1+10^{-3}\times 10\right)}{A_c\left(1+2\times 10^{-3}\times 10\right)} \times \frac{\rho_b}{\left(1+10^{-3}\times 10\right)}$$

$$P' = \frac{P}{1+2\times 10^{-2}}$$

$$\left(\frac{P'}{P} - 1\right)\times 100 = \frac{1-(1+2\times 10^{-2})}{1}\times 100 = -2\%$$

Q.10 (C)

dx = Δdx



$$\int_0^{\Delta L} \Delta dx = \int_0^{L} dx\left(3x+2\right)\times 10^{-6}\left(20-0\right)$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3x^2}{2} + 2x\right)_0^L$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3L^2}{2} + 2L\right) = 1\cdot 2 \text{ cm}$$

$L_{new} = L + \Delta L$

Q.11 (C)

Let eq^n. temp = t then

$m_R\, s_R\, t$	$= m_s\, s_s\,(100 - t)$	...(1)
$d_R{}'$	$= d_R\,(1 + \alpha_R\, t)$	...(2)
$d_s{}'$	$= d_s[1 - \alpha_s\,(100 - t)]$	...(3)
Now $d_R{}'$	$= d_s{}'$	...(4)
So. $d_R(1 + \alpha_R\, t)$	$= d_s[1 - \alpha_s(100 - t)]$	

$$t = \frac{d_s\left(1-\alpha_s 100\right) - d_R}{\left[d_R\,\alpha_R - d_s\,\alpha_s\right]}$$

Put the above value of t in eq. 1.

$$\left(\frac{m_R\, s_R}{m_S\, s_s} + 1\right)t = 100; \qquad \frac{m_s}{m_r} = \frac{23}{54}$$

Q.12 (B)

$w_1 = Mg - F_B$

$$w_2 = Mg - F_B\left[\frac{1+\gamma_m\Delta T}{1+\gamma_\ell\Delta T}\right] = Mg - F_B[1 + (\gamma_m - \gamma_\ell)\Delta T]$$

Since , $\gamma_m < \gamma_\ell$
So , $w_2 > w_1$.

Q.13 (A)

At 40°C

1 Unit will be = $1(1 + \alpha_s\,\Delta t)$ units
= $1(1 + 12\times 10^{-6}\times 40)$ Units

So 100 Unit will be = $100\,(1 + 12\times 10^{-6}\times 40)$ = Actual

$100\,(1 + 40\times 12\times 10^{-6}) = l_0\,(1 + (2\times 10^{-6})\,40)$

$l_0 = 100\,[\,1 + 400\times 10^{-6}] > 100$mm.

Q.14 (B)

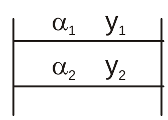
Given $\beta = 1.4\times 10^{11}$ Pa, $\alpha = 1.7\times 10^{-5}\,°C^{-1}$

ΔT = 30°C - 20°C = 10°C

$$\beta = -\frac{\Delta P}{\Delta V/V} \Rightarrow \Delta P = -\beta\,\frac{\Delta V}{V}$$

$\Delta P = \beta(3\alpha\,\Delta T)$
$= 1.4\times 10^{11}\times 3\times 1.7\times 10^{-5}\times 10$
$= 7.14\times 10^{7}$ Pa.

Q.15 (C)



$$\frac{\alpha_1}{\alpha_2} = \frac{2}{6}$$

$\because\ \dfrac{F}{A} = Y\,\alpha\,\Delta\theta \qquad \because\ \Delta\theta$ is same for both

$$\frac{\frac{F_1}{A_1}}{\frac{F_2}{A_2}} = \frac{Y_1\alpha_1}{Y_2\alpha_2}$$

$$\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = 3:1$$

Q.16 (A,B)
On heating or cooling water from 4ºC it expands in both cases.

Q.17 (A)
In bimetallic strips the two metals have different thermal expansion coefficient. Hence on heating it bents towards the metal with lower thermal expansion coefficient.

Q.18 (C)
$6 \times 10^{-5} = 1 \times 12 \times 10^{-6} \times \Delta T$

$$\frac{6\times10^{-5}}{12\times10^{-6}} = \Delta T \Rightarrow \Delta T = 5^{o}C.$$

JEE-ADVANCED
MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(ABC)**
Stress in wire B $= \dfrac{mg}{3\pi\, r_B^2}$

Stress in wire A $= \dfrac{4mg}{3\pi\, r_A^2}$

if $\dfrac{mg}{3\pi\, r_B^2} = \dfrac{4mg}{3\pi\, r_A^2}$ either wire will break.

Q.2 **(A, C, D)**
Gravitational Potential Energy $U_G = Mgl$

Elastic Potential Energy $U_e = \dfrac{1}{2}$ stress × strain × volume

$$= \frac{1}{2}\frac{F}{A}\frac{\ell}{\ell_0}\times V \left| \begin{array}{l} F = mg \\ V = A\ \ell_0 \end{array}\right.$$

$$= \frac{1}{2} mgl$$

Heat Produced $= U_e = \dfrac{1}{2} Mgl$

Q.3 **(A,B,C,D)**
On heating, every dimension increases.

Q.4. **(BC)**
Strain $\rightarrow$ Same

Stress $= \dfrac{F}{A} =$ constant

$F \propto A$
$\Rightarrow F \propto r^2$

Energy $= \dfrac{1}{2}$ stress × strain × volume

$\propto$ Area
$\propto r^2$

Q.5 **(A, C, D)**
(A) % rise in area $= \beta\Delta T = 2(\alpha\Delta T) = 2 \times 0.2 = 0.4\%$
(C) % rise is volume $= 3\,\alpha\Delta T = 3 \times 0.2 = 0.6\%$

(D) $\alpha = \dfrac{0.2}{80\times100} = 0.25 \times 10^{-4}/°C$

Q.6 (B)
Because floating

$$\rho_s Vg = \rho_\ell \left(\frac{V}{2}\right) g$$

$$2\rho_s = \rho_\ell$$

Q.7 (A)
if $\gamma_L > \gamma_S$ then submerged more else come out of liquid respectively
and $\gamma_L > \gamma_s$ (always)

Q.8 (A)
$V' = V[1 + \gamma_s \Delta t]$
$\rho_l' = \rho_l [1 - \gamma_l \Delta t]$

$$\rho_l\left(\frac{V}{2}\right) g = \rho_l'\left(\frac{V'}{2}\right) g$$

$$\rho_l\left(\frac{V}{2}\right) g = \rho_l(1-\gamma_l\Delta t)\left(\frac{V}{2}\right)(1+\gamma_s\Delta t)g$$

$(1 - \gamma_l \Delta t)(1 + \gamma_s \Delta t) = 1$
$(1 - \gamma_l \Delta t)(1 + 3\alpha_s \Delta t) = 1$
$3\alpha_s - \gamma_l = 0$

Q.9 (A)

initially $\rho_l(A_s h)g = (\rho_s A_s h_o)g$

...(1)

Now $\rho_l'(A_s' h) g = (\rho_s' A_s' h_o')g$

...(2)

$\rho_l (1 - \gamma_l \Delta t)h = \rho_s(1 - 3\alpha_s \Delta t)h_o (1 + \alpha_s \Delta t)$

$\gamma_L = 2\alpha_s$

Q.10 (A)$\rho_l' < r_s$ or $\frac{\rho_l}{2}$

$\frac{\rho_l}{1+\gamma_l \Delta t} < \frac{\rho_l}{2}$

$1 + \gamma_l \Delta t > 2$

$\Delta t > \frac{1}{\gamma_l}$

$T_F - T > \frac{1}{\gamma_l}$ $\Rightarrow T_F > T + \frac{1}{\gamma_l}$

Q.11 (C)

$K = \frac{AY}{\ell} = \frac{4\times10^{-4}\times2\times10^{10}}{4} = 2 \times 10^6$

$\omega = \sqrt{\frac{K}{m}} = 100$

Q.12 (D)

$W(h + x) = 1/2\ kx^2$

$100(0.99 + x) = \frac{1}{2} \times 2 \times 10^6 \times x^2$

$10^4 x^2 - x - 0.99 = 0$

$100\ x\ (100x - 1) + 0.99\ (100\ x - 1) = 0$

$x = \frac{1}{100}$ m = 1 cm

Q.13 (A)

$x = \frac{PL}{AY}$

$\sigma = \frac{P}{A} = \frac{xY}{L} = \frac{10^{-2} x 2 x 10^{10}}{4} = 5 \times 10^7$ N/m^2

Q.14 (C)

$K_1 = 10^6$, $K_2 = 2 \times 10^6$

$K_{eq} = \frac{2\times10^6 x 10^6}{3\times10^6} = \frac{2}{3}\times10^6$

$\omega = \sqrt{\frac{2\times10^6}{3\times600}} = \frac{100}{3}$

Q.15 (B)

Total weight = 1000 + w

weight on each rod = $\frac{1000+w}{4}$

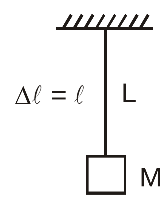
stress = $\frac{1000+w}{4\times4\times10^{-4}} = 9 \times 10^6$

$\Rightarrow$ w = 14400 – 1000 = 13400 N

No. of persons are = $\frac{1340}{50} = 26$

Q.16 (A) p (B) q ; (C) r ; (D) q

loss in PE = $Mg\ell$



Elastic PE = $\frac{1}{2}Kx^2$

$= \frac{1}{2}\frac{Mg}{A} x \frac{\ell}{L} x AL$

= MgL/2

Heat = MgL – Mg L/2

= Mg L/2

Q.17 (A) – (r) ; (B) – (q) ; (C) – (p) ; (D) – (s)

Q.18 (A) – (p) ; (B) – (r) ; (C) – (s) ; (D) – (q)

(A) Buoyant force = Mg = constant = $V_{sub}\rho_\ell \times g$

$V_{sub} = \frac{Mg}{\rho_\ell g}$.

volume of displace fluid = constant

$\therefore$ density of fluid must be constant.

(B) $(V_{solid} - V_{sub})$ = constant

$$\Rightarrow \left(V_{solid} - \frac{M_{solid}}{\rho_{liquid}}\right) = \text{constant}$$

$$V \times 3\alpha \times \Delta T = \frac{M\gamma\Delta T}{d}$$

$$\Rightarrow \gamma = 3\alpha\frac{d}{\rho}$$

(C) $Ah_{in}\, d_{liquid} = A(h_{in} + h_{out})\, \rho_{solid} = M$ (mass of solid)

$$h_{out} = \frac{M}{A\rho_{solid}} - h_{in} = \frac{M}{A\rho_{solid}} - \frac{M}{Ad_{liquid}} = \text{constant}$$

$$\frac{M(1+3\alpha\Delta T)}{A(1+2\alpha\Delta T)\rho} - \frac{M(1+\gamma\Delta T)}{A(1+2\alpha\Delta T)d} = \frac{M}{A\rho} - \frac{M}{Ad}$$

$$\gamma = 2\alpha + \alpha\frac{d}{\rho}$$

(D) $Ah_{in}\, d_{liquid}\, g$ = Buoyant force = constant = Mg

$$A_0(1 + 2\alpha\Delta T)\, h_{in} \frac{d}{1+\gamma\Delta T} = \text{constant}$$

$$h_{in} = \frac{M}{A_0 d}(1 + (\gamma - 2\alpha)\Delta T)$$

$$\gamma = 2\alpha$$

NUMERICAL VALUE BASED

Q.1 [250]

$$\frac{\Delta l}{l} = \frac{\pi\times 0.05}{\pi\times 40} = \frac{1}{800}$$

$$T = Y\frac{\Delta l}{l}\times A = 200\times 10^9 \times \frac{1}{800}\times 1\times 10^{-6}$$

= 250 N

Q.2 [1000]

Decrease in temperature would cause shrinking of wire, as wire is attached at 2 ends, this would result in tension (stress) in wire.

$\alpha = 2\times 10^{-6}$

$$\frac{F}{A} = \frac{\Delta\ell}{\ell}$$

$$\frac{\Delta\ell}{\ell} = \frac{2.2\times 10^8}{1.1\times 10^{11}} = 2\times 10^{-3}$$

$$\frac{\Delta\ell}{\ell} = \alpha\Delta T = (2\times 10^{-3}) = (2\times 10^{-6})(\Delta T)$$

$$\Delta\ell = 1000°C$$

Q.3 [500]

Temp. is increased by $\Delta\theta$ then

$$\Delta l = l\alpha\Delta\theta$$

$$\Rightarrow \Delta\theta = \frac{\Delta l}{l\alpha}$$

$$E_1 = (\rho A l)S\Delta\theta = \rho A l\, S\frac{\Delta l}{l\alpha}$$

when stretched, Stress = $Y\dfrac{\Delta l}{l}$

$$E_2 = \frac{1}{2}\left(Y\frac{\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right)\times Al = \frac{Y(\Delta l)^2 A}{2l}$$

$$\text{So, } \frac{E_1}{E_2} = \frac{\rho A l S\Delta l\times 2l}{l\times Y(\Delta l)^2 A} = \frac{2\rho S l}{\alpha(\Delta l)Y} = 500$$

Q.4 [0012]

Q.5 [5]

At equilibrium

$2T\sin\theta = mg$

$$\Rightarrow 2.\left(\frac{YA}{2a}\right) x\sin\theta.\sin\theta = mg$$

$$\Rightarrow \frac{YA}{a}\, x.\, \frac{x^2}{a^2} = mg$$

$$\Rightarrow x = \left\{\frac{a^3 mg}{YA}\right\}^{\frac{1}{3}} = \left\{\frac{1m\times 5kg\times 10\,m/s^2}{(2.4\times 10^9\,N/m^2)\times 10^{-4}m^2}\right\}^{\frac{1}{3}}$$

= 5 cm

Q.6 [2]

$$\Delta\ell = \frac{F\ell}{\Delta y}$$

$$\frac{\Delta\ell}{F/A} = \frac{\ell}{y}$$

$$y = \frac{4000\times 10^3}{2\times 10^{-3}} = 2\times 10^9\ N/m^2$$

Q.7 [4]

Stress = $\dfrac{40}{10^{-3}}$ = 4 × 10^4 N/m

Q.8 [1]

$$\left(\frac{\Delta L}{L}\right)_S = \frac{T_S}{A_S Y_S}$$

$$\left(\frac{\Delta L}{L}\right)_B = \frac{T_S}{A_B Y_B}$$

strain equal

$$\frac{T_S}{A_S Y_S} = \frac{T_B}{A_B Y_B}$$

$$\Rightarrow \frac{T_S}{0.1 \times 2 \times 10^{11}} = \frac{T_B}{0.2 \times 1 \times 10^{11}}$$

$T_S = T_B$

Take torque absent O

$T_S \times x = T_B(200 - x)$

$x = 100$ cm

Q.9 [3]

Let us calculate elongation (ξ) in part length x from lower side.

$$F = \frac{mg}{L}x$$

$$\text{Stress} = \frac{mg}{AL}x$$

$$\text{Strain } \frac{d\xi_x}{dx} = \frac{mg}{YAL}x$$

$$\int_0^{\xi_x} d\xi_x = \frac{mg}{YAL}\int_0^x x\,dx$$

$$\xi_x = \frac{mg}{2YAL}x^2$$

$$\frac{\xi_{\text{upper half}}}{\xi_{\text{lower half}}} = \frac{\dfrac{mg}{2YAl}\left\{L^2 - \left(\dfrac{L}{2}\right)^2\right\}}{\dfrac{mg}{2YAl}\left(\dfrac{L}{2}\right)^2} = \frac{3}{1}.$$

Q.10 [10000]

$$F = Ay\frac{\Delta\ell}{\ell}$$

$= Ay\ \alpha\ \Delta\theta = 10^{-3} \times 10^{11} \times 10^{-6}\ (100 - 0°)$

$= 10000$ N

KVPY

PREVIOUS YEAR'S

Q.1 (A)

Will increases by an amount $d\alpha\Delta T$

Q.2 (A)

Q.3 (D)

Increase in length of each liquid is same.

$\Delta\ell = \Delta\ell$

$$\frac{\Delta V_{Hg}}{\pi d_1^2} = \frac{\Delta V_{Bromine}}{\pi d_2^2}$$

$$\frac{(V)\gamma_{Hg}\Delta\theta}{\pi d_1^2} = \frac{\Delta V\gamma_{Bromine}\Delta\theta}{\pi d_2^2}$$

$$\left(\frac{d_1}{d_2}\right)^2 = \frac{\gamma_{Hg}}{\gamma_{Bromine}} = \frac{18\times10^{-5}}{108\times10^{-5}}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{1}{6}} \simeq 0.4$$

Q.4 (C)

When some boyd is constrained from expanding or bending then on heating thermal stress get develop in the body.

Stress = $Y\ \alpha\ \Delta T$

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times (40 - 25) = 3.3 \times 10^7$ N/m^2

$= 3.3 \times 10^7$ Pa

Q.5 (A)

Initially wire is slack so it do not have any deformation energy. When block is given some velocity it move due to kinetic energy, one wire get taut. Internal force get develop in wire and KE start decreases and deformation energy of wire increase. Till block come at rest using energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times (\text{strain})^2 \times A \times L$$

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times \left(\frac{x}{L}\right)^2 \times A \times L$$

$$x = v\sqrt{\frac{mL}{AY}}$$

Q.6 (D)

high density is not the reason for its uses in clinical thermometers.

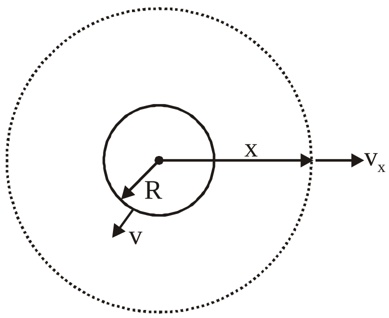
Q.7 (B)

Thermal expansion of a solid is due to asymmetric characteristic of inter atomic potential energy curve of the solid.

Q.8 (A)

Since when temperature of water rises from 0ºC to 10 ºC, its density first increase, becomes maximum at 4ºC and then decreases, therefore fractional submergence will first decrease and then increase.

Q.9 (B)



$A_1v_1 = A_2v_2$

$(4\pi R^2)v = (4\pi x^2)v_x$

$$v_x = \frac{R^2}{x^2}.v$$

so small kinetic energy at x for width $dx = dk = \frac{1}{2}\,dm\,(v_x^{\,2})$

$$dk = \frac{1}{2}[\rho(4\pi x^2 dx)v_x^{\,2}]$$

$$= \frac{1}{2}\rho 4\pi x^2\left(\frac{R^2}{x^2}v\right)^2 dx$$

Total kinetic energy

$$k = \int dk = 2\rho\pi R^4 v^2 \int_R^{\infty}\frac{dx}{x^2}$$

$$k = 2\rho\pi R^4 v^2\left[-\frac{1}{x}\right]_R^{\infty} = 2\rho\pi R^3 v^2$$

Q.10 (B)

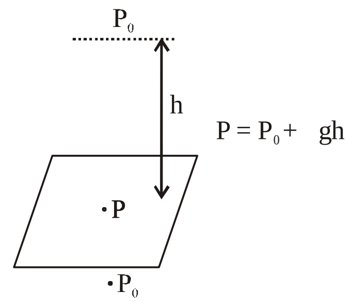
Volume of water received from rain =

$$V = 600\times10^6\times2.4\times\frac{10}{100}$$

$V = 1440\times10^5\ m^3$

$$\%\text{ of needed water } = \frac{1440\times10^5}{1.4\times10^{12}}\times100 \approx 10\%$$

Q.11 (A)



P = Pressure on upper surface of window

$= P_0 + \rho gh$

Pin = Pressure inside the submarine

$= P_0$

Net force $= (P_0 + \rho gh)A - P_0A$

$= \rho gh\ A$

$= 1.03\times10^3\times10\times100\times900\times10^{-4}$

$= 9.27\times10^4$ Newton

$= 0.93\times10^5$ Newton

Q.12 (B)

Apply Bernoulli between point-1 and point-2

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 = P_{atm} + \frac{\text{Force by hand}}{\text{Area}}$$

V_1 tends to zero becomes are of point-2 is very small.

$P_2 = P_{atm}$

$$P_{atm} + F/A = P_{at} + (1/2)\ \rho V_2^2 = \frac{2F}{PA} \quad ....(i)$$

From Kinematics.

$$2 = \sqrt{\frac{2(h)}{g}}\times V_2$$

$\therefore\ V_2^2 = 20$

Using (i) & (ii) we get ...(ii)

$$20 = \frac{2F}{\rho A} \quad \therefore F = 10\,N$$

Q.13 (A)

$$\text{Stress } = \frac{\text{Force}}{\text{Area}} \propto \frac{1}{\text{Area}}$$

Stress $\propto \frac{1}{r^2}(\text{Area} = \pi r^2)$

ratio of stress = $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (1)

$$\frac{\Delta V}{V} = \gamma\Delta T$$

$$= 3\alpha\Delta T$$

Q.2 (4)

$$\frac{T_1}{A} = \frac{y(\ell_1 - \ell)}{\ell}$$

$$\frac{T_2}{A} = \frac{y(\ell_2 - \ell)}{\ell}$$

$$\frac{T_1}{T_2} = \frac{\ell_1 - \ell}{\ell_2 - \ell}$$

$$T_1\ell_2 - T_1\ell = T_2\ell_1 - T_2\ell$$

$$\frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2} = \ell$$

Q.3 (2)

$$\frac{F}{A} = v\frac{\Delta L}{L}$$

$$\frac{F}{A} = y \times \frac{0.04}{L} \qquad \text{...(i)}$$

$$\frac{F}{4A} = y \times \frac{\Delta L}{2L} \qquad \text{...(ii)}$$

$$4 = \frac{0.04 \times 2}{\Delta L}$$

$$\Delta L = 2 \times 10^{-2}$$

$$x = 2$$

Q.4 (1)

Q.5 [32]

For A $\frac{E}{\pi r^2} = y\frac{2mm}{a}$ ....(1)

For B $\frac{E}{\pi.16r^2} = y\frac{4mm}{b}$ ....(2)

$\therefore$ (1)/(2)

$$16 = \frac{2b}{4a}$$

$$\frac{a}{b} = \frac{1}{32}$$

$\therefore$ Answer = 32

Q.6 (1)

$P_1 = \rho gd + P_0 = 3 \times 10^5$ Pa

$\therefore \rho gd = 2 \times 10^5$ Pa

$P_2 = 2\rho gd + P_0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

% increase = $\frac{P_1 - P_2}{P_1} \times 100$

$$= \frac{5\times10^5 - 3\times10^5}{3\times10^5} \times 100 = \frac{200}{3}\%$$

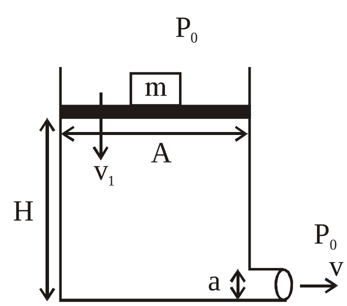
Q.7 (4)

(4) $\Delta P = h\rho g$

$$B = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{-2\times10^3\times10^3\times9.8}{\left(-1.36\times10^{-2}\right)}$$

$= 1.44 \times 10^9$ N/m^2

Q.8 (3)



m = 24 kg

A = 0.4 m^2

a = 1 cm^2

H = 40cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho gH + \cancelto{0}{\frac{1}{2}\rho v_1^2}$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \qquad \text{... (1)}$$

= Neglecting v_1

$$\Rightarrow v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow v = \sqrt{8 + 1.2}$$
$$\Rightarrow v = 3.033 \text{ m/s}$$
$$\Rightarrow v \simeq 3 \text{m / s}$$

Q.9 (1)

Q.10 (2)

Q.11 [5]

Q.12 (2)

Q.13 [20]

Q.14 (2)

$$Y = \frac{MgL^3}{4bd^3\delta}$$

$$\frac{\Delta y}{y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta}$$

$$\frac{\Delta y}{y} = \frac{10^{-3}}{2} + \frac{3\times10^{-3}}{1} + \frac{10^{-2}}{4} + \frac{3\times10^{-2}}{4} + \frac{10^{-2}}{5}$$

$= 10^{-3}\,[0.5 + 3 + 2.5 + 7.5 + 2] = 0.0155$

Option (2)

Q.15 [40]

Q.16 (3)

Q.17 (4)

Q.18 (2)

Force on each column = $\frac{mg}{4}$

Strain = $\frac{mg}{4AY}$

$$= \frac{50\times10^3\times9.8}{4\times\pi(1-0.25)\times2\times10^{11}} = 2.6\times10^{-7}$$

Q.19 [500]

Q.20 [8]

Thermal force $F = Ay\propto\Delta T$

$F = (10\times10^{-4})\,(2\times10^{11})\,(10^{-5})\,(400)$

$F = 8\times10^5$ N

$\Rightarrow x = 8$

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 [3]

$$\frac{F}{A} = y\,\frac{\Delta L}{L}$$

$$\frac{mg}{A} = y\times(\alpha\Delta\theta)$$

m =

$$\frac{Ay\alpha(\Delta\theta)}{g} = \frac{\pi r^2 y\alpha(\Delta\theta)}{g} = \frac{\pi(10^{-3})^2\times10^{11}\times10^{-5}\times10}{10}$$

$= \pi \approx 3$

Q.2 (C)

$$Y = \frac{\left(\frac{F}{A}\right)}{\frac{\Delta\ell_1}{L}} \qquad \text{...(i)}$$

$$Y = \frac{\left(\frac{F}{4A}\right)}{\frac{\Delta\ell_2}{2L}} \qquad \text{...(ii)}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = 2$$

Q.3 [0.23 to 0.24] [2.38]

$$\frac{dV}{V} = \frac{3da}{a}$$

$$B = -V\frac{dP}{dV} = \frac{-V(\rho gh)}{dV} = \frac{-\rho gh}{3da}a$$

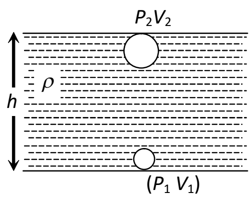
$$70\times10^9 = \frac{1\times5000\times10^3\times10\times1}{3\times da}$$

$$da = \Delta a = \frac{5}{21}\times10^{-2}\text{ m} = 2.38\text{mm}$$

ELEMENTARY

Q.1 (2)

According to Boyle's law, pressure and volume are inversely proportional to each other i.e. $P \propto \frac{1}{V}$



$\Rightarrow P_1V_1 = P_2V_2$

$\Rightarrow (P_0 + h\rho_w g)V_1 = P_0V_2$

$$\Rightarrow V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$$

$$\Rightarrow V_2 = \left(1 + \frac{47.6 \times 10^2 \times 1 \times 1000}{70 \times 13.6 \times 1000}\right)V_1$$

$\Rightarrow V_2 = (1 + 5)\ 50\ cm^3 = 300\ cm^3.$

[As $P_2 = P_0 = 70$ cm of Hg $= 70 \times 13.6 \times 1000$]

Q.2 (3)

As the both points are at the surface of liquid and these points are in the open atmosphere. So both point possess similar pressure and equal to 1 atm. Hence the pressure difference will be zero.

Q.3 (3)

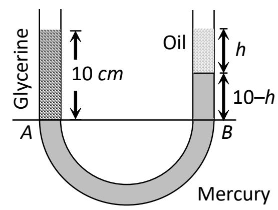
$P_1V_1 = P_2V_2 1. \Rightarrow (P_0 + h\rho g)\ V = P_0 \times 3V$

$$\Rightarrow h\rho g = 2P_0 \Rightarrow h = \frac{2 \times 75 \times 13.6 \times g}{\frac{13.6}{10} \times g} = 15m$$

Q.4 (4)

Pressure = $h\rho g$ i.e. pressure at the bottom is independent of the area of the bottom of the tank. It depends on the height of water upto which the tank is filled with water. As in both the tanks, the levels of water are the same, pressure at the bottom is also the same.

Q.5 (4)



At the condition of equilibrium

Pressure at point A = Pressure at point B

$P_A = P_B \Rightarrow 10 \times 1.3 \times g = h \times 0.8 \times g + (10 - h) \times 13.6 \times g$

By solving we get h = 9.7 cm

Q.6 (4)

Let M_0 = mass of body in vacuum.

Apparent weight of the body in air = Apparent weight of standard weights in air

$\Rightarrow$ Actual weight – upthrust due to displaced air

= Actual weight – upthrust due to displaced air

$\Rightarrow$ 1

$\Rightarrow$

$$M_0 g - \left(\frac{M_0}{d_1}\right)dg = Mg - \left(\frac{M}{d_2}\right)dg \Rightarrow M_0 = \frac{M\left[1 - \frac{d}{d_2}\right]}{\left[1 - \frac{d}{d_1}\right]}$$

Q.7 (4)

Apparent weight

$= V(\rho - \sigma)g = l \times b \times h \times (5 - 1) \times g$

$= 5 \times 5 \times 5 \times 4 \times g$ Dyne $= 4 \times 5 \times 5 \times 5$ gf.

Q.8 (1)

Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right)V$

i.e. it depends upon the densities of the block and liquid. So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

Q.9 (2)

$$V\rho g = \frac{V}{2}\sigma g$$

$$\therefore\ \rho = \frac{\sigma}{2}(\sigma = \text{density of water})$$

Q.10 (2)

For streamline flow, Reynold's number $N_R \propto \frac{r\rho}{\eta}$ should be less. For less value of N_R, radius and density should be small and viscosity should be high.

Q.11 (3)
If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.
$\therefore M = m_1 + m_2 \Rightarrow A\nu_1 = A\nu_2 + 1.5A.\nu$
$\Rightarrow A \times 3 = 4 \times 1.5 + 1.5A.\nu \Rightarrow \nu = 1$ m/s

Q.12 (4)
As cross-section areas of both the tubes A and C are same and tube is horizontal. Hence according to equation of continuity $\nu_A = \nu_C$ and therefore according to Bernoulli's theorem $P_A = P_C$ i.e. height of liquid is same in both the tubes *A* and C.

Q.13 (2)
Bernoulli's theorem for unit mass of liquid

$$\frac{P}{\rho} + \frac{1}{2}\nu^2 = \text{ constant}$$

As the liquid starts flowing, it pressure energy decreases

$$\frac{1}{2}\nu^2 = \frac{P_1 - P_2}{\rho} \Rightarrow \frac{1}{2}\nu^2 = \frac{3.5\times10^5 - 3\times10^5}{10^3} \Rightarrow \nu^2$$

$$= \frac{2\times0.5\times10^5}{10^3} \Rightarrow \nu^2 = 100 \Rightarrow \nu = 10 \text{ m / s}$$

Q.14 (1)

Pressure at the bottom of tank $P = h\rho g = 3\times10^5 \frac{N}{m^2}$

Pressure due to liquid column
$P_l = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$

and velocity of water $\nu = \sqrt{2gh}$

$$\therefore \nu = \sqrt{\frac{2P_l}{\rho}} = \sqrt{\frac{2\times2\times10^5}{10^3}} = \sqrt{400} \text{ m/s}$$

Q.15 (4)
Upthrust – weight of body = apparent weight
$VDg - Vdg = Vd\alpha$,

Where *a* = retardation of body $\therefore \alpha = \left(\frac{D-d}{d}\right)g$

The velocity gained after fall from h height in air,

$\nu = \sqrt{2gh}$

Hence, time to come in rest,

$$t = \frac{\nu}{\alpha} = \frac{\sqrt{2gh}\times d}{(D-d)g} = \sqrt{\frac{2h}{g}} \times \frac{d}{(D-d)}$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (3)
rg(H – h)
because pressure varies with height.

Q.2 (1)
F = [rgh] [A]
= (1000) (10) (6) (10) (8).

Q.3 (2)
$W_A > W_B$ as mass of water in A is more than in B
$P_A = P_B$
Area of A = Area of B
or $P_A \text{Area}_A = P_B \text{Area}_B$
or $F_A = F_B$.

Q.4 **(2)**
Pressure = $h\rho g_{eff.}$
a = g/3
$g_{eff} = g - g/3 = 2g/3$

$$\Rightarrow P = \frac{0.15\times1000\times2\times10}{3}$$

P = 1KPa

Q.5 (2)
Given $A = 2 \times 10^{-3}$, h = 0.4 m, r = 900 Kg/m^3
$F = mg = Vrg = (pr^2h)rg$
$= 2 \times 10^{-3} \times 0.4 \times 900 \times 10$
= 7.2 N

Q.6 (1)
F = mg
F = 10 N

Q.7 (4)
O(zero) all the forces passes through O
$\therefore$ no torque.

Q.8 **(A)**

Q.9 (4)

$$Dv = v_f - v_i = \frac{m}{y} - \frac{m}{x}.$$

Q.10 (2)

$h \rho g = 2P$

$\frac{4h}{5} \rho g = \frac{8P}{5}$ $\left\{ \begin{array}{c} \text{After lowering P due} \\ \text{to liquid.} \end{array} \right\}$

$\therefore P_T = \frac{8P}{5} + P$ (Atmospheric pressure)

$= \frac{13P}{5}$

Q.11 (1)

At same depth pressure is same

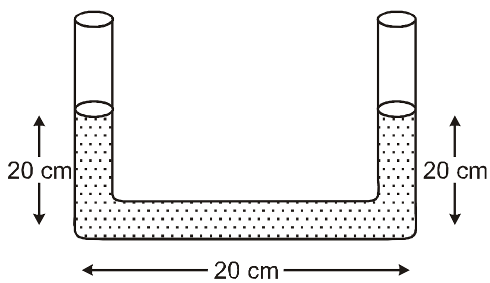
So ratio $P_1 : P_2 = 1 : 1$.

Q.12 (1)

$\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2}$

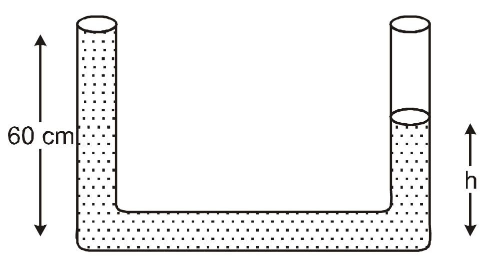
Solving, $m_2 = 3.75$ kg.

Q.13 (D)



$60 \; r_w g = h \; r_l g$

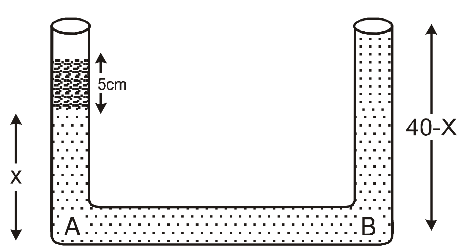
$\Rightarrow 60 \times 1 \times g = h \times 4 \Rightarrow h = 15$ cm



So, volume = Ah

$= 1 \times 35 = 35$ cm^3

Q.14 (3)



$P_A = P_B$

$\Rightarrow 5 \times 4 \times g + x \times 1 \times g$

$= (40 - x) \times 1 \times g$

$\Rightarrow x = 10$

Now, $h_1 = x + 5 = 15$ cm

$h_2 = 40 - x = 30$ cm

$h_2/h_1 = 2$

Q.15 (3)

Given m = 12 kg, A = 800 cm^2, r = 1000 kg/m^3

P = rgh

$\frac{mg}{A} = rgh$

$\frac{12 \times 10}{800 \times 10^{-4}} = 1000 \times 10 \times h$

$\frac{12}{80} = h$

$h = \frac{1200}{80} = 15$ cm

Q.16 (2)

$F_b = rVg - rvg = 0$

Q.17 **(1)**

mg = 60

.................(i)

$mg - r_l vg = 40$

.................(ii)

$\frac{mg - \rho_\ell v g}{mg} = \frac{2}{3}$ or $\frac{\rho_0}{\rho_\ell} = 3$

where r_0 = density of the block and r_l = density of the liquid.

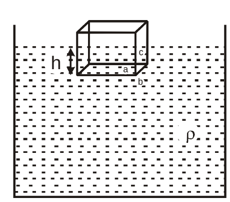
Q.18 (3)

$10^3 \times \frac{4}{5} + 13.5 \times 10^3 \times \frac{1}{5} = r \times 1$

or $r = 3.5 \times 10^3$ kg/m^3

Q.19 (2)

Q.20 (D)



At equilibrium position

(abc) (dr)g = (bc) hrg

After displacing slightly x,

extra buoyancy force.

$r_{net} = ((bc)x)rg$

$a = \frac{xbc\rho g}{abcd\rho} = \frac{xg}{ad}$ $\qquad w = \sqrt{\frac{g}{ad}}$

Q.21 (C)

$[36 - r_1 v_1]g = [48 - r_1 v_2]g$

$$\left[36 - \rho_i\left(\frac{36}{9}\right)\right]g = \left[48 - \rho_i\left(\frac{48}{\rho_0}\right)\right]g$$

Solving, $r_0 = 3$.

Q.22 (1)

In stable equilibrium the object comes to its original state if disturbed.

Q.23 (3)

As, weight = Buoyant force

$mg = [100 \times 6 \times 0.6\ g] + (100 \times 1 \times 4)g$

$\therefore \quad m = 760$ gm.

Q.24 (2)

$r_1 V = r_2 2V$

$m_1 = m_2$

$m_1 g = 0.92\ Vg = m_2 g - xVg$

$x = 1.8\ gm/cm^3$

Q.25 (3)

Q.26 (2)

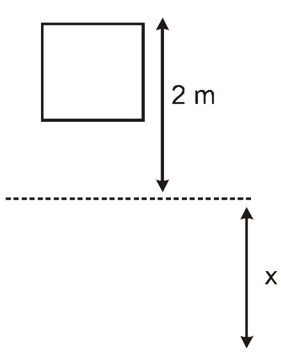
$W - v \times 1 \times g = W_1$

$W - v \times x \times g = W_2$

$\Rightarrow W - (W - W_1) \times x = W_2$

$x = \frac{W - W_2}{W - W_1}$

Q.27 (A)



$mg\ (x + 2) = v \times 1 \times g \times x$

$v\ 0.8\ g\ (x + 2) = v \times 1 \times g \times x$

$0.8\ x + 1.6 = x$

$0.2x = 1.6$

$x = 8$

Q.28 (4)

Equilibrium Position $W = F_B$

$W = L^2 h\ r_M g$

$h = \frac{W}{L^2 \rho_M g}$

Q.29 (3)

$\therefore$ Volume where metal is present

$= \frac{9.8}{7800} = 1.256 \times 10^{-3}$

Buoyancy = vrg = 1.5 g

$\Rightarrow v \times 1000 = 1.5$

$v = 1.5 \times 10^{-3}$

fraction of volume =

$\frac{1.5\times10^{-3} - 1.256\times10^{-3}}{1.5\times10^{-3}} \times 100$

$= 16\%$

Q.30 (1)

$\sigma \times \frac{4}{3}\pi (R^3 - r^3)g = 1 \times \frac{4}{3}\pi R^3 g$

$\frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$

Q.31 (2)

Volume $= \frac{0.5}{500} = 10^{-3} m^3$

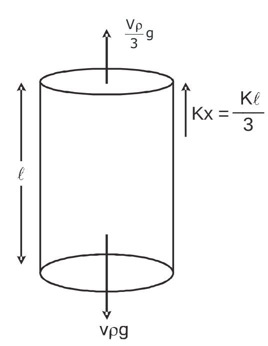
Buoyancy $= \rho Vg = 1000 \times 10^{-3} \times 10$

$= 10$ N

$m = 1$ kg

If float = 2.5 kg, Reading = 1 + 1.5 = 2.5 kg

Q.32 (2)

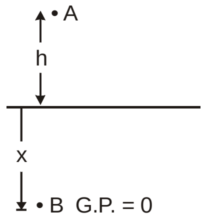


$V = A.\ell.$

Now $\frac{A\ell\rho g}{3} + \frac{K\ell}{3} = A\ell\rho$

$K = 2\rho Ag$

Q.33 (C)



$W_{FB} = \{K_B + U_B\} - \{K_A + U_A\}$

$-V\rho x g = 0 + 0 - 0 - V\rho' g(x + h)$

$\rho g x = \rho' g x + \rho' g h$

$$x = \frac{\rho' h}{\rho - \rho'}$$

Q.34 (2)

Apparent weight ($W_{app.}$) = $W - V\rho_\ell g$

Since, $W_{app.\,(Ram)} > W_{app.\,(Shyam)}$

$\Rightarrow \; W_{(Ram)} > W_{(Shyam)}$

Therefore, from given passage shyam has more fat than Ram.

Q.35 (2)

$V_1 > V_2 \Rightarrow W_{app.\,(1)} < W_{app.\,(2)}$

(Since $W_{app.} = W - V\rho_\ell g$)

Hence (2)

Q.36 (1)

$\rho_{\text{Salt waver}} > \rho_{\text{Fresh waver}} \quad \Rightarrow W_{app.\,(s)} < W_{app.\,(F)}$

Hence (1)

Q.37 (3)

Let 'V' be the total volume of the person

Then ;

$$\left(\frac{V}{4}\right)(0.4 \times 10^3) + \left(\frac{3}{4}V\right)\left(\frac{4}{3}\times 10^3\right) = 165$$

$$\Rightarrow \; V = \frac{165}{1100}$$

Reading on spring balance under water is :

$$W_{app} = [165 \times 10] - \left[\frac{165}{1100}\right][10^3][10]$$

$$= 150\,\text{N}$$

Q.38 (4)

Just after the string is cut :

$$a = \frac{150}{165} = 0.91\ \text{m/s}^2$$ **Ans.**

Q.39 (3)

$R = vt$

$$= \sqrt{2gD}\,\sqrt{\frac{2(H - D)}{g}}$$

$$= 2\sqrt{D(H - D)}\,.$$

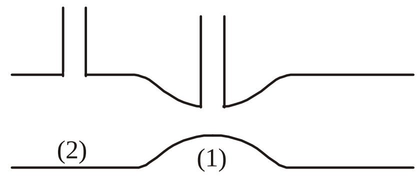
Q.40 (3)

$x = 2\sqrt{h(H - h)}$

for x_{max}, $\dfrac{dx}{dh} = 0$ or $h = \dfrac{H}{2}$

Q.41 (1)

From continuity equation, velocity at cross-section (1) is more than that at cross-section (2).



Hence ; $P_1 < P_2$

Q.42 (2)

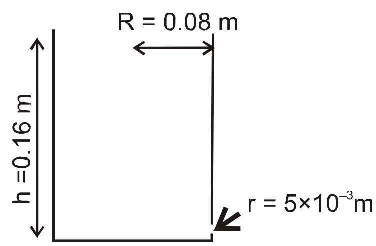
$F_{thrust} = \rho a v^2$

$F_{net} = F_1 - F_2 = a\rho[2g(h_1 - h_2)]$

$= a\rho(2gh)$

or $\quad F \propto h$

Q.43 (1)



$A_1 v_1 = A_2 v_2$

$\pi R^2\, dh/dt = \pi r^2 v$

....(i)

$v = \sqrt{2gh}$

....(ii)

from equation (ii) put the value of v in equation (i)

$$\pi R^2\, dh/dt = \pi r^2\sqrt{2gh}$$

$$\Rightarrow \int \frac{R^2 dh}{r^2\sqrt{2gh}} = \int dt$$

$$\frac{R^2}{r^2\sqrt{2g}}\int_h^0 \frac{dh}{\sqrt{h}} = \int_0^t dt$$

on solving

$t = 46.26$ second.

Q.44 (1)

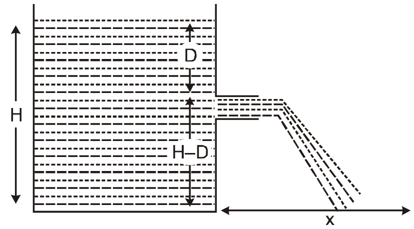
AV = constant

$A\downarrow V\uparrow$

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

$V\uparrow P\downarrow$

Q.45 (A)



$$\Delta\rho g = \frac{1}{2}\rho v^2$$

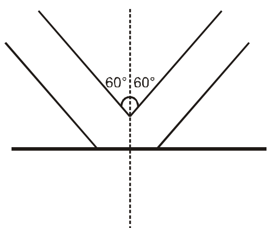
$$v = \sqrt{2Dg}$$

$$x = V\sqrt{\frac{2(H-D)}{g}} = 2\sqrt{D(H-D)}$$

Q.46 (B)

$$\frac{dm}{dt} = \rho A v$$

$\Delta P = F_{avg}$. 1sec.

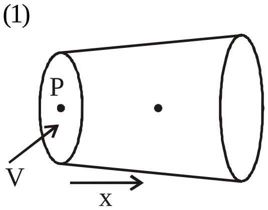


$\Rightarrow 2\rho A v^2 \cos 60°$

$\Rightarrow 1000 \times 6 \times 10^{-4} \times (12) \times \frac{1}{2} \times 2$

$= 86.4$ Nt.

Q.47 (1)



$A = ax + b$

Continuty equation $bV = (ax + b)\, V_2$

By bernaulies equation $= P_2 + \frac{1}{2}\rho v_2^2 =$ cosntant

$P_2 = \text{Costant} - \frac{1}{2}\rho v_2^2$

$P_2 = \text{Costant} - \frac{1}{2}\rho \frac{b^2V^2}{(ax+b)^2}$

$P_2 = \text{Cosntant} - \frac{C_1}{(ax+b)^2}$

Where C_1 = Constant

Q.48 (2)

$A_1V_1 = A_2V_2$

$0.02 \times 2 = 0.01 \times V_2$

$V_2 = 4$ m/sec.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$4 \times 10^4 + \frac{1}{2} \times 1000 \times 2^2$$

$$= P_2 + \frac{1}{2} \times 1000 \times 4^2 \Rightarrow P_2 = 3.4 \times 10^4 \text{ N/m}^2$$

Q.49 (2)

$\sqrt{2 \times 20 \times 10^{-2} \times 10} = 2$ m/sec.

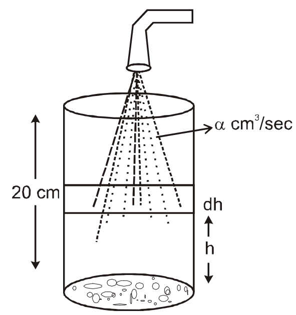
Q.50 (2)

Q.51 (4)

Inlet = outlet

$\alpha\, dt = a\sqrt{2gh}\, dt$

$$h = \frac{\alpha^2}{2ga^2} = \frac{(100)^2}{2 \times (1000)(1)} = 5 \text{ cm}$$

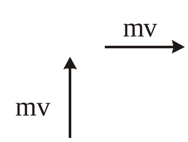


Q.52 (4)

Force exerted by the water on the corner

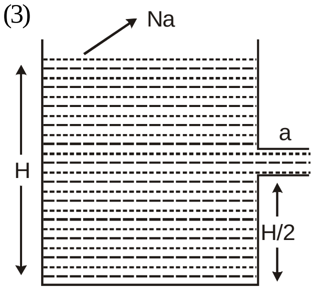
= change in momentum in 1 sec

$= \sqrt{2}$ mv



$= \sqrt{2}\rho v L$

Q.53 (3)



Force = $\rho a\left(\sqrt{2gh/2}\right)^2$

acceleration = $\dfrac{\rho agh}{\rho Na.H}$ = g/N

Q.54 (2)

$\alpha dt = Av\,dt$

$\Rightarrow 10^{-4} = 10^{-4}\sqrt{2gh} \Rightarrow h = \dfrac{1}{2g}$

h = 0.051 m

Q.55 (D)

$R = \sqrt{2g(H-h_1)}\sqrt{\dfrac{2h_1}{g}}$

$= \sqrt{2g(H-h_2)}\sqrt{\dfrac{2h_2}{g}}$

$(H-h_1)\,h_1 = (H-h_2)\,h_2$

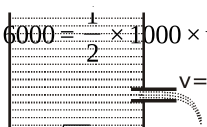
$H = h_1 + h_2$

For max. range = $\dfrac{H}{2}$

Q.56 (2)

By Bernaulie's Theorem

$P_0 + \dfrac{10\times10}{1000\times10^{-4}} + 1000\times10\times\dfrac{50}{100} = P_0 + \dfrac{1}{2}\times1000\times v^2$



$6000 = \dfrac{1}{2}\times1000\times v^2$

$v = \sqrt{12} = 3.4$ m/s

Q.57 (1)

Change in momentum is/sec.

$\sqrt{2}\,\rho Av^2 = 565.7$ N.

Q.58 (2)

$\rho AV^2 = 1000\times2\times10^{-4}\times(10)^2$

= 20 N

Q.59 (3)

Energy required in one second is the power

$10^{-1} = A.V.$

$\Rightarrow 10^{-1} = 10^{-2}\times V$

$\Rightarrow V = 10$ m/sec.

$mgh + \dfrac{1}{2}mV^2 = P$

Here m = mass in one second

$P = \rho AVgh + \dfrac{1}{2}\rho AV^3$

$P = \rho AV[10\times10+50]$

= 15 Kwatt

Q.60 (3)

Q.61 (3)

If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.

$\therefore M = m_1 + m_2 \Rightarrow A\nu_1 = A\nu_2 + 1.5A.\nu$

$\Rightarrow A\times3 = 4\times1.5 + 1.5A.\nu \Rightarrow \nu = 1$ m/s

Q.62 (2)

$A_1V_1 = A_2V_2$ (Given $\dfrac{r_1}{r_2} = \dfrac{3}{2}$)

$\dfrac{v_1}{v_2} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

Q.63 (4)

$A_1V_1 = A_2V_2$

$\pi(1\times10^{-2})^2\times3$

$= 100\times\pi\dfrac{\left(0.05\times10^{-2}\right)^2}{4}\times V_2 \Rightarrow V_2 = 48$ m/sec.

Q.64 (1)

From $A_PV_P = A_QV_Q$

$\dfrac{V_P}{V_Q} = \dfrac{A_Q}{A_P} = \dfrac{\pi(2\times10^{-2})^2}{\pi(1\times10^{-2})^2}$

$V_P = 4V_Q$

Q.65 (3)

$\dfrac{dV}{dt} = A\sqrt{2gh}$

Q.66 (1)

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (C)

Since not touching,

So $R = F_b = \rho_l(vg) = 40g$.

$R' - R = 80g - 40g = 40g$

Hence R′ will be 40g more than R

Q.2 (A)

By action – reaction, F_b is internal

So, balance weight = $(m_1 + m_2)g$.

and extra mass = 10 g

Q.3 (D)
Force is same pressure is different

Q.4 (B)
Take area of projection from left

$$\frac{2\rho g\ell h^2}{2} = \frac{3\rho g\ell R^2}{2}$$

$$h = \sqrt{\frac{3}{2}}.R$$

Q.5 (B)
Pressure exerted by fluid at closed end B is

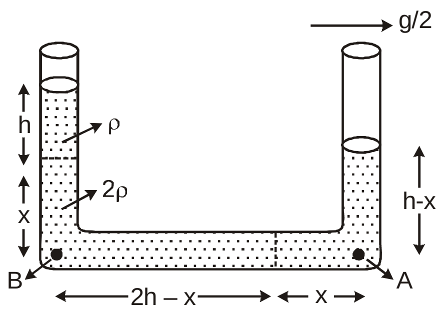
$P = \rho g\ell$

$\therefore$ force exerted by fluid at closed end B is

$F = PA = \ell\rho g A_0$

Q.6 (A)

Q.7 (B)



$\rho xg/2 + 2(2h\text{-}x)g/2 = P_B - P_A$

$P_A = (h - x)\rho g$

$P_B = h\rho g + 2\rho xg$

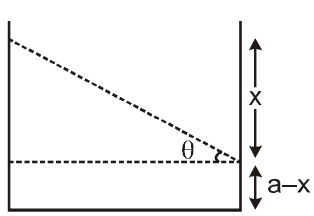
$\rho xg/2 + 2\rho(2h–x)g/2 = h\rho g + 2\rho xg–h\rho g + x\rho g$

$$\frac{4\rho hg}{2} - \frac{2\rho xg}{2} = 2\rho xg + \rho xg/2$$

$2\rho hg - \rho xg = 5Pxg/2$

$$\Rightarrow x = \frac{4h}{7}$$

Q.8 (B)



$$(a\text{-}x)a^2 + \frac{1}{2}\,x\,a^2 = \frac{2}{3}a^3$$

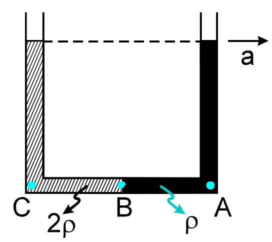
$$(a\text{-}x) + \frac{x}{2} = \frac{2}{3}a$$

$$a - \frac{x}{2} = \frac{2}{3}a$$

$$x = \frac{2a}{3}$$

$$\therefore \tan\theta = \frac{x}{a} = \frac{acc.}{g} \quad a = \frac{2}{3}g$$

Q.9 (B)
For the given situation, liquid of density 2 ρ should be behind that of ρ.



From right limb :

$P_A = P_{atm} + \rho gh$

$$P_B = P_A + \rho a\frac{\ell}{2} = P_{atm} + \rho gh + \rho a\frac{\ell}{2}$$

$$P_C = P_B + (2\rho)a\frac{\ell}{2} = P_{atm} + \rho gh + \frac{3}{2}\rho a\ell$$

.... (1)

But from left limb :

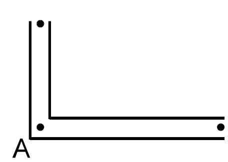
$P_C = P_{atm} + (2\rho)gh$

.... (2)

From (1) and (2) :

$$P_{atm} + \rho gh + \frac{3}{2}\rho a\ell = P_{atm} + 2\rho gh \Rightarrow h = \frac{3a}{2g}\ell \quad \textbf{Ans.}$$

Q.10 (A)
No sliding $\Rightarrow$ pure rolling
Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')



$P_A = P_{atm} + \rho(2a)L$ (From horizontal limb)

Also ; $P_A = P_{atm} + \rho g H$ (From vertical limb)

$$\Rightarrow a = \frac{gH}{2L} \quad \textbf{Ans.}$$

Q.11 (B)

As long as $\rho \le \rho_W$, pressure at the bottom of the pan would be same everywhere, according to the Pascal's law.

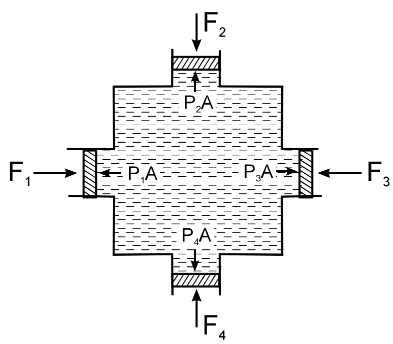
Q.12 (B)

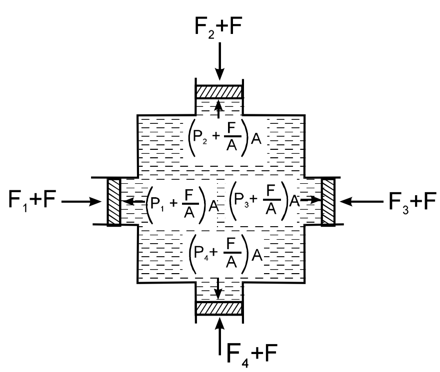
$$y = \frac{\omega^2 r^2}{2y}$$

Put values and get y = 2cm.

Q.13 (A)

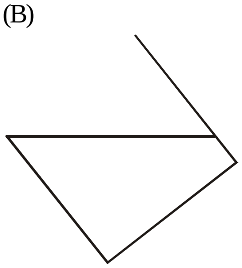
The four piston are initially in equilibrium. As additional force F is applied to each piston, the pressure in fluid at each point must be increased by $\frac{F}{A}$ so that each piston retains state of equilibrium.

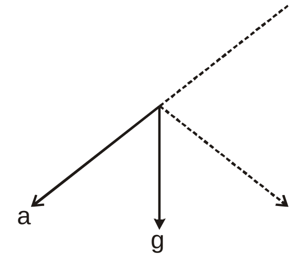




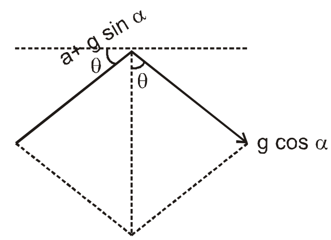
Thus the increment in pressure at each point is $\Delta P = \frac{F}{A}$ (by Pascal's law)

Q.14 (B)

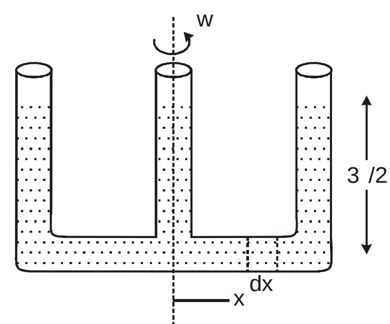




$$\tan q = \frac{a + g\sin\alpha}{g\cos\alpha}$$



Q.15 (C)



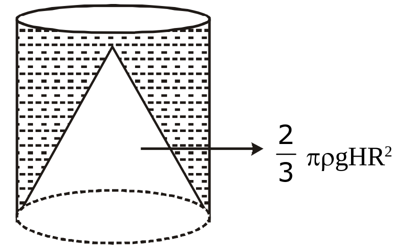
dP . A = r A dx . w^2x

$$\int dp = \int \rho\omega^2 x dx$$

$$DP = \frac{\rho\omega^2\ell^2}{2} = \frac{3\ell\rho g}{2}$$

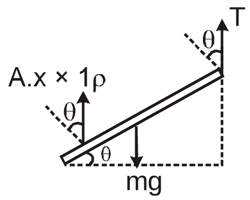
$$w = \sqrt{\frac{3g}{\ell}}$$

Q.16 (D)



Q.17 (A)

Q.18 (A)



$mg = A.2L \times 0.75 \times g$

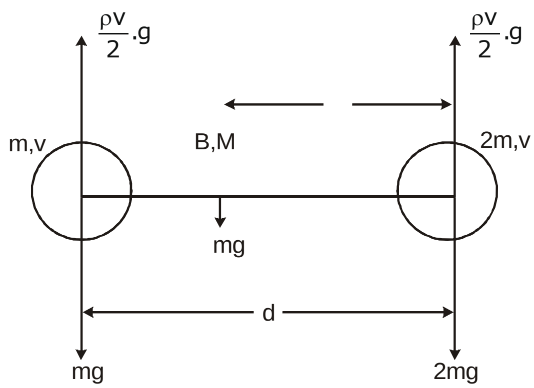
$T + Axg = A.2L \times 0.75\, g$

$T = Ag\,[1.5\,L - x]$

$$A\,xg\cos\theta\left(\ell - \frac{x}{2}\right) = T\cos\theta\,\ell$$

$$x\left(\ell - \frac{x}{2}\right) = \ell\,[1.5\,L - x]\; x = \ell$$

Q.19 (B)



$(3M + m)g = \rho vg$

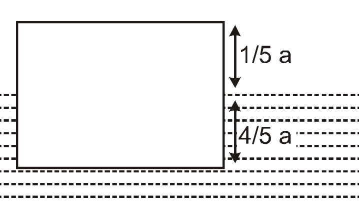
Torque balance about B

$$mg\,(d-\ell) + \rho vg\,\frac{\ell}{2}$$

$$= 2\,mg\ell + \frac{\rho vg}{2}(d - \ell) \qquad \ell = \frac{d(v\rho - 2M)}{2(\rho v - 3M)}$$

Q.20 (C)

$\rho.\ \dfrac{4}{5}a^3 = M$



$(m + M) = \rho a^3$

$m = \dfrac{5}{4}\ M\ m = \dfrac{M}{4}$

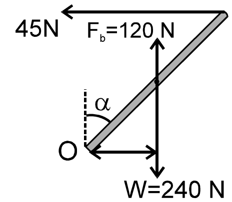
$M = 4\,m$

Q.21 (B)

F.B.D. of rod :

$W = (0.012)(1)(2 \times 10^3)(10) = 240\,N$

$F_b = (0.012)(1)(10^3)(10) = 120\,N$



Torque about O

(For equilibrium)

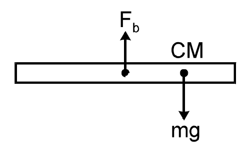
$$(240 - 120)\left(\frac{\sin\alpha}{2}\right) = 45\,(\cos\alpha) \Rightarrow \tan\alpha = \frac{90}{120}$$

$$= \frac{3}{4} \Rightarrow \alpha = 37°$$

Q.22 (B)

Torque about CM :

$$F_b \cdot \frac{\ell}{4} = I\alpha$$

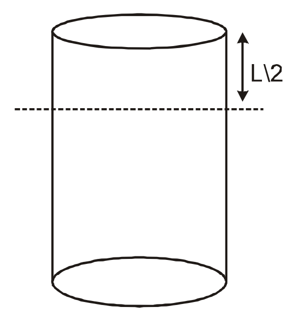


$$\Rightarrow \quad \alpha = \frac{1}{I}(\pi r^2)(\ell)(\rho)(g).\frac{\ell}{4}\ \alpha = \frac{\pi r^2 \ell^2 g\rho}{4I}$$

'α' will be same for all points

Hence (B).

Q.23 (A)

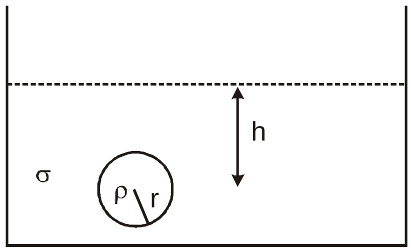


$$d_1AL + d_2AL = \frac{3}{2}\,LAd$$

$$d_1 + d_2 = \frac{3d}{2}$$

$$\therefore\ d_1 > \frac{3d}{4}$$

Q.24 (B)



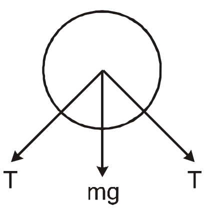
$(W.D.)_{mg} + (W.D.)_{FB} = \Delta K$
$-mg\,(H + h) + (F_B)h = 1/2\, mv_f^2$

$$\Rightarrow\ -\frac{4}{3}\pi r^3 \rho\,(H + h)\,g + \left(\frac{4}{3}\pi r^3\right)\sigma gh = 0$$

$-\rho gH - \rho gH + \sigma gh = 0$
$gh\,(\sigma - \rho) = \rho gH$

$$\boxed{H = \left(\frac{\sigma}{\rho} - 1\right)h}$$

Q.25 (A)

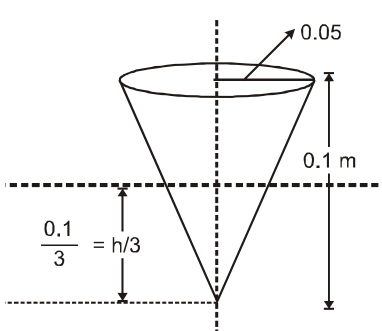


$$\frac{2T}{\sqrt{2}} + mg = v\rho_w g$$

Q.26 (B)
$g_{eff} = g + a$
$\Rightarrow T + mg_{eff} = F_B$
$T = Vd(g + a) - v\rho\,(g+a)$
$= v[(g + a)\,(d-\rho)]$

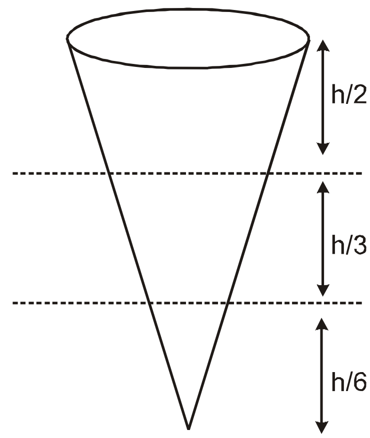
Q.27 (A)
Increasing the temperature of water from 2^0C to 3^0C increases its density while decreases the density of iron.
Hence the bouyant force increases.

Q.28 (D)



$$\frac{1}{3}\pi r^2 h\rho_c g = \frac{1}{3}\pi(r/3)^2\,\frac{h}{3}\,(0.8)g$$

$$\rho_c = \frac{0.8}{27}$$



$$\frac{1}{3}\pi r^2 h\rho_c g = \frac{1}{3}\pi\left(\frac{r}{6}\right)^2\frac{h}{6} \times 0.8 \times g +$$

$$\frac{1}{3}\pi\left[\left(\frac{r}{2}\right)^2\frac{h}{2} - \left(\frac{r}{6}\right)^2\frac{h}{6}\right]\rho g$$

$$\Rightarrow \frac{0.8}{27} = \frac{0.8}{36\times 6} + \left[\frac{1}{8} - \frac{1}{36\times 6}\right]\rho \quad \rho = 1.9$$

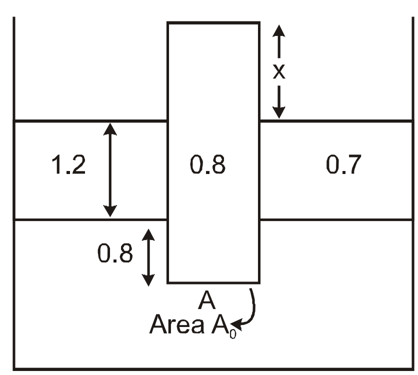
Q.29 (B)
Initially
$W_{metal} = W_{ice}$ = Buoyancy
$V_{metal}\,\rho mg + V_{ice}\,\rho_{ice}\,g = V_d\rho_\ell g$

$$\therefore V_d = \frac{v_{metal}\ \rho_m}{\rho_\ell} + \frac{V_{ice}\rho_{ice}}{\rho_\ell}$$

finally volume displaced
$V = V_m + V_w$ (From ice)

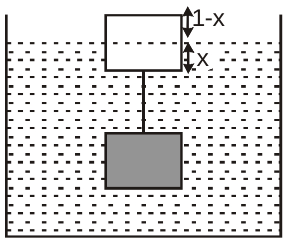
$$= V_m + \frac{m}{\rho_w} = V_m + \frac{V_i\ell_i}{\rho_w} < \text{previous}$$

Q.30 (B)



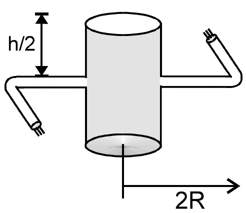
$P_A = (1.2 \times 0.7 \times g + 0.8 \times 1.2\, g)$
$0.8 \times A_0 (x + 1.2 + 0.8)\, g = P_A.P_0$
$x + 1.2 + 0.8 = 0.84 + 0.96$
$x = 0.25$ cm

Q.31 (B)



Total buoyancy
= Total Gravitation
$\Rightarrow 1^3 \times 1 \times g + 1^2 x \times 1r$
$= 1^3 \times 0.6 \times g + 1^3\, 1.15 \times g$
$1 + x = 0.6 + 1.15$
$x = 0.75$ m
$\therefore\ 1 - x = 25$ cm.

Q.32 (A)



Velocity of efflux of water (v) $= \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

force on ejected water = Rate of change of momentum of ejected water.
$= \rho\,(av)\,(v)$
$= \rho\, av^2$
Torque of these forces about central line
$= (\rho a v^2)\, 2R\, .\, 2$
$= 4\rho a v^2 R = 4\rho\, agh\, R$

Q.33 (B)

$A_1V_1 = A_2V_2$ or $A \times V_1 = 2A\sqrt{\frac{g\ell}{2}}$ or $V_1 = \sqrt{2g\ell}$

$\frac{1}{2}\rho\left[V_1^2 - V_2^2\right] = \rho g\ell \sin\theta \Rightarrow \frac{1}{2}\left[2g\ell - \frac{g\ell}{2}\right] = g\ell \sin\theta$

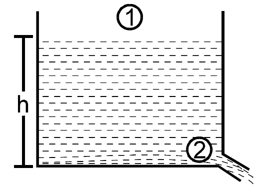
on solving $\sin\theta\ \frac{3}{4}$.

Q.34 (D)

The velocity of fluid at the hole is : $V_2 = \sqrt{\frac{2gh}{1+(a^2/A^2)}}$

Using continuity equation at the two cross-sections (1) and (2) :

$V_1A = V_2a \qquad \Rightarrow \quad V_1 = \frac{a}{A}V_2$



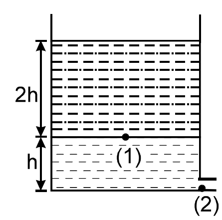
$\Rightarrow$ acceleration (of top surface) $= -V_1\frac{dV_1}{dh}$

$= -\frac{a}{A}V_2\frac{d}{dh}\left(\frac{a}{A}V_2\right)$

$a_1 = -\frac{a^2}{A^2}V_2\frac{dV_2}{dh} = -\frac{a^2}{A^2}\sqrt{2gh}\sqrt{2g}.\frac{1}{2\sqrt{h}} \Rightarrow a_1 = \frac{-ga^2}{A^2}$

Q.35 (B)
Pressure at (1) :
$P_1 = P_{atm} + \rho g\,(2h)$
Applying Bernoulli's theorum between points (1) and (2)

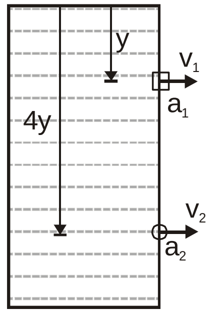
$[P_{atm} + 2\rho g h] + \rho g(2h) + \frac{1}{2}(2\rho)(0)^2$



$= P_{atm} + (2\rho)\, g\,(0) + \frac{1}{2}(2\rho)\, v^2$

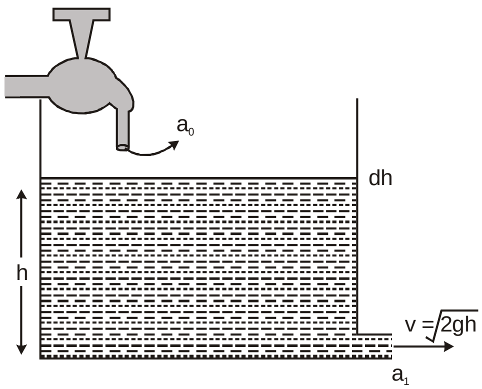
$\Rightarrow\ v = 2\sqrt{gh}$ **Ans.**

Q.36 (A)
Velocity of efflux at a depth h is given by V = Volume of water following out per second from both the holes are equal
$\therefore\ a_1V_1 = a_2V_2$
or $(L^2)\sqrt{2g(y)} = \pi R^2\sqrt{2g(4y)}$



or $R = \dfrac{L}{\sqrt{2\pi}}$

Q.37 (C)



$$\frac{dm}{dt} = \rho A\frac{dh}{dt} = \rho a_0 v - \rho a_1\sqrt{2gh}$$

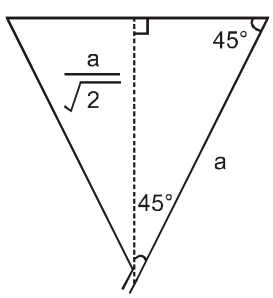
$$\Rightarrow 4000\frac{dh}{dt} = 1 \times 2 - 0.5\sqrt{2gh}$$

$$\text{for } t = \infty \quad \frac{dh}{dt} = 0$$

$$\Rightarrow 2 = 0.5\sqrt{2gh}$$

$$\Rightarrow h = 0.8$$

Q.38 (D)



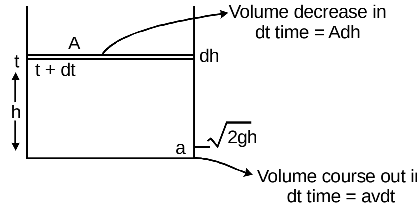
Initial

$$a\rho g = \frac{1}{2}\rho V^2$$

$$V_0 = \sqrt{2ga}$$

$$\text{Now } V = \sqrt{2\frac{a}{\sqrt{2}}} = \frac{V_0}{\sqrt[4]{2}}$$

Q.39 (C)



Volume decrease = Volume outlet

$$Adh = a\sqrt{2gh}\,dt$$

$$\frac{-dh}{dt} = \frac{a\sqrt{2gh}}{A} \Rightarrow \int_H^{H/\eta}\frac{dh}{\sqrt{2gh}} = -\int_0^t \frac{a}{A}dt$$

$$t_1 = \left[-\sqrt{\frac{H}{\eta}} + \sqrt{H}\right]$$

$$\text{Similarly } t_2 = \sqrt{\frac{H}{\eta}}$$

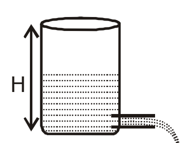
$$\Rightarrow t_1 = t_2 \Rightarrow 2\sqrt{\frac{H}{\eta}} = \sqrt{H}$$

$$\eta = 4$$

Q.40 (A)

Q.41 (D)
We know that

$$t_0 = \sqrt{\frac{2H}{g}}$$



When height become 4H then time

$$t' = \sqrt{2\frac{(4H)}{g}}$$

$$t' = 2t_0$$

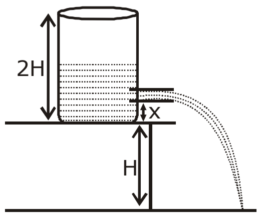
Q.42 (C)

$R = vt$

$v = \sqrt{2(2H - x)g}$

$t = \sqrt{\dfrac{2(H + x)}{g}}$

$R_{max} = \dfrac{dR}{dx} = 0$



& we get

$x = \dfrac{H}{2}$

Total height from ground $= H + \dfrac{H}{2} = 1.5\,H$

Q.43 (D)

$R = \sqrt{2g \times 10}\sqrt{\dfrac{2H}{g}}$.

.... (1)

Now $\rho gh + P_O + P_E = P_O + \dfrac{1}{2}\rho V^2$

$\Rightarrow V^2 = 2gh + \dfrac{2P_E}{\rho}$

$\Rightarrow R' = \sqrt{(2g10) + (\dfrac{2P_E}{\rho})}\sqrt{\dfrac{2H}{g}}$

.....(2)

From (1) & (2) $P_E = 3$ atm.

Q.44 (D)

$v_1 = \sqrt{2gh/2} = \sqrt{gh}$

for v_2

$\rho gh + 2\rho g\dfrac{h}{2} = \dfrac{1}{2}\,2\rho.v_2^2$

$2gh = v_2^2$

$v_2 = \sqrt{2gh}$

Q.45 (C)

$A_1V_1 = A_2V_2$

$10^{-2} \times 2 = 0.5 \times 10^{-2} \times V^2$

$V_2 = 4$ m/sec.

$P_A + \dfrac{1}{2}\rho V_A^2 = P_B + \dfrac{1}{2}\rho V_B^2$

$8000 + \dfrac{1}{2}\,1000 \times 2^2$

$= P_B + \dfrac{1}{2}\,1000 \times 4^2$

$P_B = 2000$ Pa

Q.46 (A)

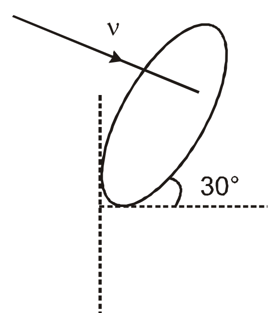
$\mu\,mg = \dfrac{2}{2}\rho\pi\left(\dfrac{d}{2}\right)^2 .2gH.$

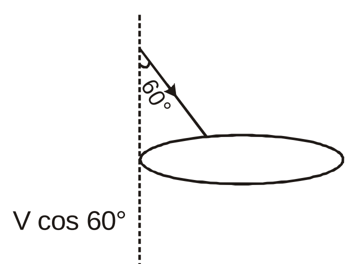
$d = \sqrt{\dfrac{2\mu M}{\pi\rho H}}$

Q.47 (C)

From A_1V_1

Where $V_1 \perp$ to area





ratio $= \dfrac{V}{V\cos 60°} = 2$

Q.48 (B)

From $A_1V_1 = A_2V_2$

$(1)(V_1)(\dfrac{1}{2})V_2 \Rightarrow \dfrac{V_1}{V_2} = \dfrac{1}{2}$

$V_2 = 2V_1$

Now,

$V_2^2 = V_1^2 + 2gh$

$4V_1^2 = V_1^2 + 2(10)\left(\dfrac{10}{100}\right)$

$V_1 = \sqrt{\frac{2}{3}}$

Now volumetric rate of flow

$= A_1V_1$

$= \frac{1\times10^{-4}}{10^{-3}} \times \frac{60\sqrt{2}}{\sqrt{3}} \quad = 4.9$ lit/min.

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, C)

In a static fluid, pressure remains same at the same level, ie, pressure do not vary with x-coordinate. Hence (C).

Q.2 (A, C, D)

P = r(2h)g

$\frac{F}{A_2}$ = r (2h) g

F_{base} = 2h r g A_2

F_{wall} = h r g [$A_2 - A_1$], at the level x

Q.3 (C, D)

Let completely submerged in water, then

$F_b = 1000 > mg(920)$ So, not possible

Let complete in oil

$F_b = (0.6)(4)(1000 + (1)(6)(100) = 840$

$F_b < mg$ So, not possible

So, let 'x' part in oil and remaining in water

$920 = [(1)(10 - x) + (0.6)(x)]\,100$

$9.2 = 10 - x + 0.6x$

$0.4x = 0.8$

x = 2 cm.

Q.4 (B, C)

PV = constant

(Assumed isothermal process)

Q.5 (A)

(A) As, $dm = A\rho_W v\,dt$

$\Rightarrow \frac{dm}{dt} = A\rho_W v$

$\Rightarrow \frac{dm}{dt} = V\rho_W \pi \frac{D^2}{4}$

where 'D' is the diameter of stream.

Q.6 (D)

$V_1A_1 = V_2A_2$

$\frac{\pi v_0 {D_0}^2}{4} = \frac{\pi v D^2}{4} \Rightarrow D = D_0\sqrt{\frac{v_0}{v}}$.

Q.7 (B)

$v = \sqrt{2gh} = \sqrt{2g(b+x)}$.

Q.8 (A)

Applying continuity equation at points with diameter D_0 & D :

$= \sqrt{2gb}.\left[\frac{\pi.{D_0}^2}{4}\right] = \sqrt{2g(b+x)}\left[\frac{\pi D^2}{4}\right]$

$\Rightarrow D = D_0\left[\frac{b}{b+x}\right]^{1/4}$

Q.9 (B)

Solving the preceding formula for the tank height h gives :

$h = x(D/D_0)^4/(1-(D/D_0)^4) = x\,D^4/({D_0}^4 - D^4)$

substituting the given parameter values gives

$h = (0.3)(0.009^4)/(0.01^4 - 0.009^4) = 0.57$ m

So the height of the water above the tap is 0.57 m or 57 cm.

One way of measuring a person's body fat content is by "weighing" them under water. This works because fat tends to float on water as it is less dense than water. On the other hand muscle and bone tend to sink as they are more dense. Knowing your "weight" under water as well as your real weight out of water, the percentage of your body's volume that is made up of fat can easily be estimated. This is only an estimate since it assumes that your body is made up of only two substances, fat (low density) and everything else (high density). The "weight" is measured by spring balance both inside and outside the water. Quotes are placed around weight to indicate that the measurement read on the scale is not your true weight, i.e. the force applied to your body by gravity, but a measurement of the net downward force on the scale.

Q.10 A - p ; B - q ; C - t ; D - s

Pressure varies with height $\Rightarrow P = \rho gh$

and is horizontal with acceleration $\Rightarrow P = \rho\ell a$

so on (A) ρgh part is zero while average force of ρax is

$\left[\frac{0+\rho\ell a}{2}\right][\ell^2]$

$= \frac{\ell\rho a}{2}(\ell^2) = \frac{(\rho\ell^3)}{2}a = \frac{ma}{2}$

In (B) $\rho\ell a$ part is zero while average force of ρgx is

$\left[\frac{0+\rho g\ell}{2}\right]\left[\ell^2\right] = \frac{\rho g}{2}(\ell^3)$

$= \frac{\rho(\ell^3)}{2}$ (g) $= \frac{ma}{2}$

Similarly for other part.

Q.11 A – q ; B - p ; C - r ; D - s

(A) On ABCD avg pressure = $\left[\frac{0+\rho_1 gh}{2}\right]$

So $F = \left[\frac{\rho_1 gh}{2}\right][\ell h] = \frac{\rho_1 gh^2\ell}{2}$

(B) No contact of ρ_2 and not any pressure on ABCD due to ρ_2

(C) On CDEF due to ρ_1, at every point pressure is $\rho_1 gh$ so average is also $\rho_1 gh$

so $F = (\rho_1 gh)(h\ell) = \rho_1 gh^2\ell$

(D) On CDEF force due to liquid of density ρ_2 is $\frac{[\rho_2 gh^2\ell]}{2}$

NUMERICAL VALUE BASED

Q.1 [0800]

In both cases, Weight = Bouyant force

Initially, $\rho_b Vg = \rho_w\left(\frac{2}{3}V\right)g \Rightarrow \rho_b = \frac{2}{3}\rho_w$

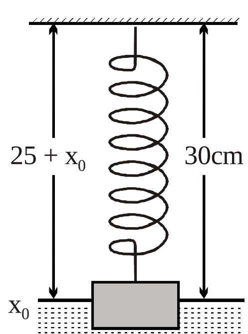
After wards, $\rho_b Vg = \rho_{oil}\left(\frac{5V}{6}\right)g$

$\Rightarrow \frac{2}{3}\rho_w = \rho_{oil} \times \frac{5}{6}$

$\Rightarrow \rho_{oil} = \frac{4}{5}\rho_w = \frac{4}{5} \times 100 = 800$ kg/m^3.]

Q.2 [5]

$30 - (25 + x_0) = 5 - x_0$



$V = \frac{32}{0.4} = 80$ cc

$A = 16$ cm^2

$kx_0 + 10^3 \times 16 \times 10^{-4} \times x_0 \times 10 = 32 \times 10^{-3} \times 10$

$x_0(48 + 16) = 32 \times 10^{-2}$

$x_0 = \frac{32}{64}$ cm = 5 mm

Q.3 [400]

$P_L \times 6 \times 10^2$ g = 600 g

mg + 600 g = $P_L \times 1000$ g

m = 1000 – 600 = 400 gm

Q.4 [250]

$\left[\frac{(500g)}{1.5\times 0.8}\right] = \frac{F}{0.06}$

$\Rightarrow$ F = 250 N

Q.5 [0006]

Mg = mg + B

$Mg = mg + p_2 \times \frac{M}{p_1} g$

$M\left(1 - \frac{p_2}{p_1}\right) = m = 6$ kg

Q.6 [720]

$(\pi R^2 H - \frac{2}{3}\pi R^3) \times d \times g$

$\pi R^2\left[H - \frac{2}{3}R\right] \times 10^4$

$= \pi \times 0.09 \times 10^4\left[1 - \frac{2}{3}\times 0.3\right]$

$\pi \times 900 \times 0.8 = 720\pi$

Q.7 [40]

$mg = \rho_1 \times 0.5 V_0 g \Rightarrow \rho_1 = \frac{2m}{V_0}$

$mg = \rho_2 \times \frac{V_0}{3} g \Rightarrow \rho_2 = \frac{3m}{V_0}$

$\rho = \frac{\rho_1 V + \rho_2 V}{2V} = \frac{\rho_1 + \rho_2}{2}$

$$mg = \rho Vg = \frac{(\rho_1+\rho_2)Vg}{2}$$

$$\Rightarrow \quad m = \frac{1}{2}\left(\frac{2m}{V_0}+\frac{3m}{V_0}\right)V$$

$$m = \frac{5mV}{2V_0}$$

$V = 0.4\,V_0 \qquad \Rightarrow \qquad 40\%]$

Q.8 [6]

$A_1v_1 = A_2v_2$

$3 \times 30 = N \times 3 \times 10^{-7} \times 0.05$

$$\frac{3\times10^8}{0.05} = N$$

$N = 6 \times 10^9$

Q.9 [2375]

$A_1v_1 = A_2v_2$

$10 \times 5 = 5 \times v_2$

$v_2 = 10$ m/s

$$\frac{p_1}{pg} + \frac{v_1^2}{2g} = \frac{v_2}{pg} + \frac{v_2^2}{2g}$$

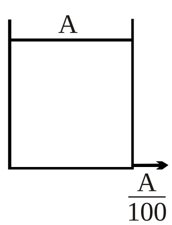
$$\frac{p_1}{10^4} + \frac{25}{20} = \frac{2\times10^5}{10^4} + \frac{100}{20}$$

$$\frac{p_1}{10^4} = 25 - 1.25 = 23.75$$

$p_1 = 2375 \times 10^2$ Pa]

Q.10 [100]

$v = \sqrt{2gh}$



$$\frac{dp}{dt} = \frac{dm}{dt}v$$

$$= \rho\,\frac{A}{100}v^2$$

$$= \frac{\rho A}{100} \times 2gh$$

$$p = \frac{\rho A}{100} \times 2g \int hdt = \frac{\rho A}{100} \times 2g \int h \times \frac{dt}{dh} \times dh$$

$$\frac{dh}{dt} \times A = \sqrt{2gh} \times \frac{A}{100}$$

$$\frac{dh}{dt} = \frac{\sqrt{2gh}}{100}$$

$$p = \frac{\rho A}{5}\,100 \int \frac{h}{\sqrt{2gh}}\,dh = \frac{\rho A}{\sqrt{20}} \times 20 \int \sqrt{h}\;dh =$$

$$\frac{\rho A}{\sqrt{20}} \times \frac{H^{3/2}}{3} \times 2$$

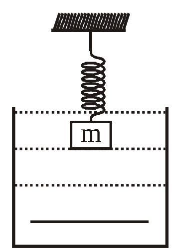
$$= \rho A\sqrt{20} \times \frac{2}{3}H^{3/2}$$

$$= 10^3 \times 3 \times 2\sqrt{5} \times \frac{2}{3} \times 5\sqrt{5} = 10^5 = 100 \text{ kNs}$$

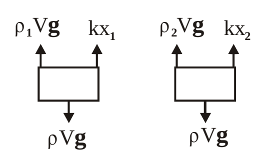
KVPY

PREVIOUS YEAR'S

Q.1 (B)



F.B.D



$kx_1 + \rho_1 Vg = \rho Vg$ ....(1)

$kx_2 + \rho_2 Vg = \rho Vg$ ....(2)

from (1) and (2)

$$\rho = \frac{\rho_2 x_1 - \rho_1 x_2}{x_1 - x_2} = \frac{\rho_1 x_2 - \rho_2 x_1}{x_2 - x_1}$$

Q.2 (B)

buyount force B = V $\rho_l g$, $\frac{V_{cube}}{V_{sphere}} = \frac{a^3}{\frac{4}{3}\pi R^3}$

but it is given $6a^2 = 4\pi R^2$

so, $\frac{V_{cube}}{V_{sphere}} = \frac{\sqrt{\pi}}{\sqrt{6}}$

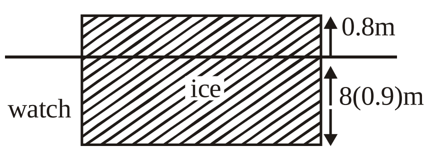
Q.3 (A)
All are nearly at same height hence $P_1 = P_2 = P_3$

Q.4 (D)
In an evacuated chamber, in absence of air, buoyancy force due to air on box is absent.

Q.5 (B)
According to Bernoulli theorem
In the region of narrow cross section of pipe, KE of fluid will be greater and pressure energy will be lesser.
$\Rightarrow$ less pressure results into larger in size of air bubble and greater KE results its greater speed.

Q.6 (D)
Blood pressure is gauge pressure = 190 mm Hg
Atmospheric pressure = 760 mm Hg
Actual pressure = 190 + 760 mm Hg = 950 mm Hg = 1.25 × 760 mm Hg

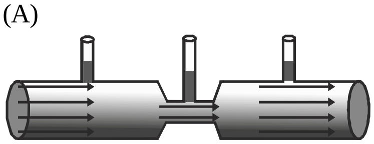
Q.7 (C)
Since $\rho_i = 0.9\ \rho_w$



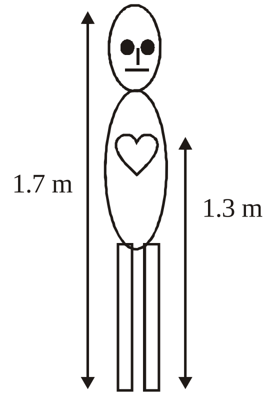
Minimum Length required = 0.8m.

Q.8 (D)
Since bucket and water both are in state of free fall so water will not come out of the hole.

Q.9 (A)



Q.10 (C)



Pressure at the heart level = 100 mm of Hg = 13.3 kPa (given)

$P_{foot} = P_{heart} + \rho gh$
$= 13.3 + 10^3 \times 10 \times 1.3 = 26.3$ kPa

$P_{head} = P_{heart} - \rho gh$
$= 9.3$ kPa

$$\frac{P_{foot}}{P_{head}} = \frac{26.3}{9.3} \approx 3$$

Q.11 (C)

$$R = V_e\sqrt{\frac{2H}{g}} = \sqrt{2gh}\sqrt{\frac{2H}{g}} = 2\sqrt{hH}$$

So velocity $V = -\frac{dR}{dt} = -2\sqrt{H}\frac{d\sqrt{h}}{dt}$

$$V = -\ 2\sqrt{H}\frac{1}{2\sqrt{h}}\frac{dh}{dt} = -\sqrt{\frac{H}{h}}\frac{dh}{dt} \quad ....(1)$$

Now AV_e = Rate of flow of volume

$$A\sqrt{2gh} = A_0\left[\frac{-dh}{dt}\right] \quad ....(2)$$

from (1) and (2)

$$V = \sqrt{\frac{H}{h}}\frac{A}{A_0}\sqrt{2gh} = \frac{1}{100}\sqrt{2\times10\times5} = \frac{1}{10}\ \text{m/s}$$

Q.12 (D)

$$\tau_1 = \int_0^h \rho g(h-x)[Ldx].x = \rho gL\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^h$$

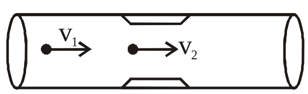
$$= \rho g L \frac{h^3}{6}$$

$$\tau_2 = \int_0^{h/2} \rho g\left(\frac{h}{2} - x\right)\left[\frac{L}{2}dx\right]x = \rho g \frac{L}{2}\left[\frac{hx^2}{4} - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \rho g \frac{L}{2} h^3 \left[\frac{1}{16} - \frac{1}{24}\right] = \frac{\rho g L h^3 \times 8}{2 \times 16 \times 24} = \frac{\rho g L h^3}{16 \times 6}$$

So, $\frac{\tau_1}{\tau_2} = 16$

Q.13 (B)



Using equation of continuity
$A_1V_1 = A_2V_2$
where A_1 & A_2 are cross-section area of region I & region-II.
as $A_2 < A_1$
$\Rightarrow V_2 > V_1$
Using Bernouilli.s equation

$$P + \frac{1}{2}\rho V^2 = \text{constant}$$

as $V_2 > V_1$
$P_2 < P_1$
therefore pressure will be lower at constriction.

Q.14 (B)
Let mass of each coin be m.
$\therefore$ Location of center of mass after N coins are kept on lid from bottom of container is

$$\frac{40m \times 0 + Nm \times 9}{(40 + N)m} = \frac{9N}{40 + N} \text{cm}$$

Also height of submerged portion after keeping N coin on lid will be,

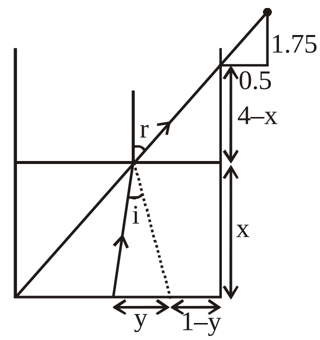
$$\frac{3(40 + N)}{40} \text{cm}$$

$\therefore$ Equilibrium will just be stable if

$$\frac{3}{40}\frac{(40 + N)}{2} = \frac{9N}{(40 + N)}$$

$\Rightarrow 3N2 - 480\,N + 4800 = 0 \Rightarrow N = 10.72$

Q.15 (C)



$1.33 \sin i = \sin r = \frac{2}{\sqrt{53}}$ ....(i)

Also, $\tan r = \frac{2}{7} = \frac{1 - y}{4 - x} \Rightarrow y = \frac{2x - 1}{7}$ .....(ii)

$\therefore$ From equation (i)

$$\frac{1.33y}{\sqrt{y^2 + x^2}} = \frac{2}{\sqrt{53}} \Rightarrow (1.33)^2 53y^2 = 4(4x^2 + y^2)$$

$\Rightarrow 89.7517\,y^2 = 4x^2 \Rightarrow y = \frac{2x}{\sqrt{89.7517}}$ ....(iii)

From equation (ii) & (iii),

$$\frac{2x - 1}{7} = \frac{2x}{\sqrt{89.517}} \Rightarrow 14x = (2x - 1)9.47$$

$\therefore x = 1.92$
$\therefore$ volume of water filled $= pR^2x$
$= (3.14 \times 1^2 \times 1.92)\ m^3$
$\therefore$ Qt = 6.0288 [Q is volume flow rate]
$\therefore$ t = 60.288 sec
so option C is the nearest value

JEE-MAIN PREVIOUS YEAR'S

Q.1 (2)
Stress is developed only if the expansion is hindered.

Q.2 [25600]

Initially $\frac{100g}{A_1} = \frac{mg}{A_2}$ .....(i)

Initially $\frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)}$ .....(ii)

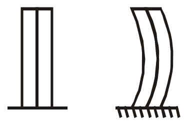
$$\frac{100 \times 16}{M} = \frac{1}{16} = M = 25600\text{kg}$$

Q.3 (1)

$\alpha_A > \alpha_B$

Length of both strips will decrease

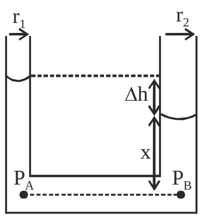
$\Delta L_A > \Delta L_B$



Q.4 (3)

Q.5 [6]

Q.6 (2)



We have $P_A = P_B$. [Points A & B at same horizontal level]

$$\therefore P_{atm} - \frac{2T}{r_1} + \rho g(x + \Delta h) = P_{atm} - \frac{2T}{r_2} + \rho g x$$

$$\therefore \rho g \Delta h = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= 2 \times 7.3 \times 10^{-2}\left[\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}}\right]$$

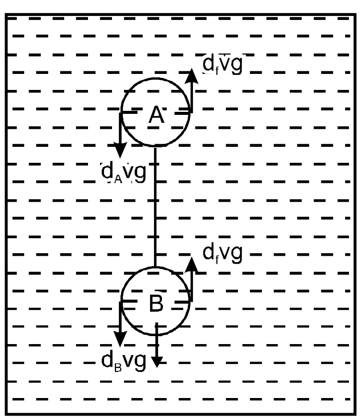
$$\therefore \Delta h = \frac{2 \times 7.3 \times 10^{-2} \times 10^{3}}{10^{3} \times 10}\left[\frac{1}{2.5} - \frac{1}{4}\right]$$

$= 2.19 \times 10^{-3}\,\text{m} = 2.19\,\text{mm}$

Hence option (2)

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 (A), (B), (D)



For equilibrium

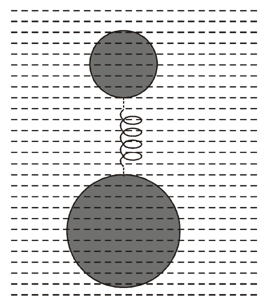
$d_A vg + d_B vg = d_F vg + d_F vg$

$\Rightarrow d_F = \dfrac{d_A + d_B}{2}$ $\Rightarrow$ Option (D) is correct

to keep the string tight

$d_B > d_F$ and $d_A < d_F$

Q.2 (A,D)



On small sphere

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g$$

..(i)

on second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx$$

...(ii)

by equation (i) and (ii)

$$x = \frac{4\pi R^3 \rho g}{3k}$$

Comprehension (Q. No. 3 to 4)

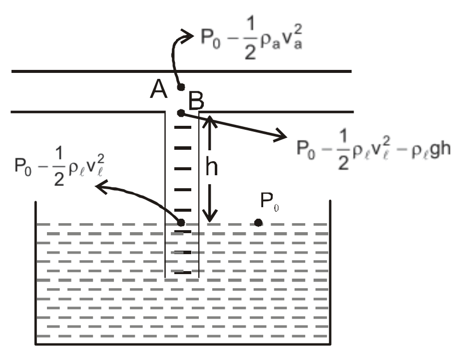
Q.3 (C)

$A_1 V_1 = A_2 V_2$ $\qquad A_1 = 400\,A_2$

$400\,(5 \times 10^{-3}) = V_2$ $\qquad \Rightarrow \boxed{V_2 = 2\,\text{m/s}}$ (C)

Q.4 (A)

Pressure at A and B will be same



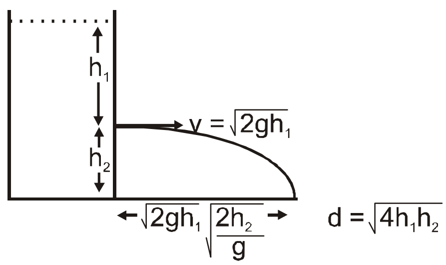
$$P_0 - \frac{1}{2}\rho_a v_a^2 = P_0 - \frac{1}{2}\rho_\ell v_\ell^2 - \rho_\ell gh$$

$$v_\ell = \sqrt{\frac{\rho_a}{\rho_\ell} v_a - 2gh}$$

Q.5 (C)

Match the column

When lift is at rest:



(P) $g_{eff} > g$

$d = \sqrt{4h_1h_2} = 1.2$ m

(Q) $g_{eff} < g$

$d = \sqrt{4h_1h_2} = 1.2$ m

(R) $g_{eff} = g$

$d = \sqrt{4h_1h_2} = 1.2$ m

(S) $g_{eff} = o$

No water leaks out of the jar.

(C) P—1; Q—1; R—1; S—4

Q.6 (B)

$h_1+h_2=0.29\times2+0.1$

$h_1+h_2=0.68$ ....(1)

$\Rightarrow P_0+\rho_k g(0.1)+\rho_w g(h_1-0.1)$[$\rho_k$=density of kerosene & ρ_w=density of water]$-\rho_w g h_2=P_0$

$\Rightarrow \rho_k g(0.1)+\rho_w g h_1-\rho_w g\times(0.1)$

$= \rho_w g h_2$

$\Rightarrow 800\times10\times0.1+1000\times10\times h_1$

$-1000\times10\times0.1=1000\times10\times h_2$

$\Rightarrow 10000(h_1-h_2)=200$

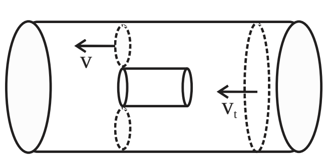
$\Rightarrow h_1-h_2=0.02$ .....(2)

$\Rightarrow h_1=0.35$

$\Rightarrow h_2=0.33$

So, $\dfrac{h_1}{h_2}=\dfrac{35}{33}$

Q.7 [9]



with respect to train

Applying Bernoulli's equation

$$P_0+\frac{1}{2}\rho v_t^2 = P+\frac{1}{2}\rho v^2$$

$$P_0-P=\frac{1}{2}\rho\left(v^2-v_t^2\right) \quad \text{....(i)}$$

From equation of continuity

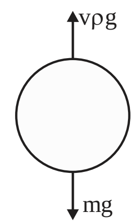
Also, $4S_t v_t = v\times3S_t \Rightarrow v=\dfrac{4}{3}v_t$ ....(ii)

From (i) and (ii)

$$P_0-P=\frac{1}{2}\rho\left(\frac{16}{9}v_t^2-v_t^2\right)=\frac{1}{2}\rho\frac{7v_t^2}{9}$$

$\therefore$ N=9

Q.8 [4]



$480\times g = v\rho_1 g$

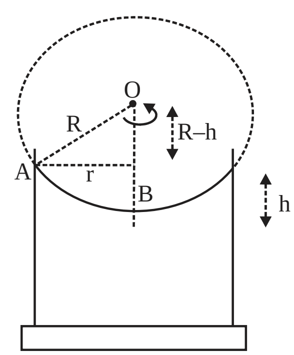
$(480-N)g = v\rho_2 g$

$$\frac{480-N}{480}=\frac{\rho_2}{\rho_1}$$

$$\left(1-\frac{N}{480}\right)=\frac{e^{-h_2/h_0}}{e^{-h_1/h_0}}=e^{\frac{h_1-h_2}{h_0}}=e^{\frac{50}{6000}}$$

$$1-\frac{N}{480}=1-\frac{50}{6000}\Rightarrow N=\frac{50\times480}{6000}=4$$

Q.9 (A,D)



In ΔOAB

$R^2=(R-h)^2+r^2$

$R^2=R^2-2hR+h^2+r^2 \qquad \Rightarrow 2hR=h^2+r^2$

$$\Rightarrow R=\frac{h^2+r^2}{2h}$$

Now considering equation of surface

$$y = y_0 + \frac{\omega^2 r^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$

Now using : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1}{v} + \frac{4}{3(H-h)} = \frac{1-4/3}{-R} \qquad \Rightarrow \frac{1}{v} = \frac{1}{3R} - \frac{4}{3H}$$

$$\Rightarrow \frac{1}{v} = \frac{2h}{3r^2} - \frac{4}{3H} \qquad \Rightarrow \frac{1}{v} = -\frac{4}{3H}\left[1 - \frac{\omega^2 H}{4g}\right]$$

$$\Rightarrow v = \frac{3H}{4}\left[1 + \frac{\omega^2 H}{4g}\right]^{-1}$$

Q.10 (AC)

Q.11 (0.30)

Q.12 (10.00)

Surface Tension and Viscosity

ELEMENTRY

Q.1 (1)

Q.2 (1)

Q.3 (1)
Weight of spiders or insects can be balanced by vertical component of force due to surface tension.

Q.4 (4)
$T = T_0(1 - \alpha t)$

Q.5 (1)
Energy needed = Increment in surface energy
= (surface energy of *n* small drops) – (surface energy of one big drop)
$= n4\pi r^2T - 4\pi R^2T = 4\pi T(nr^2 - R^2)$

Q.6 (3)
Work done to increase the diameter of bubble from d to D

$$W = 2\pi\left(D^2 - d^2\right)T = 2\pi\left[(2D)^2 - (D)^2\right]T = 6\pi D^2T$$

Q.7 (3)

$$W = 8\pi T(r_2^2 - r_1^2) = 8\pi T\left[\left(\frac{2}{\sqrt{\pi}}\right)^2 - \left(\frac{1}{\sqrt{\pi}}\right)^2\right]$$

$$\therefore\ W = 8\times\pi\times 30\times\frac{3}{\pi} = 720 \text{ erg}$$

Q.8 (1)

Q.9 (3)

Q.10 (2)

Q.11 (3)
Angle of contact is acute.

Q.12 (3)

Since $\Delta P \propto \dfrac{1}{R}$

Q.13 (2)

Q.14 (1)

$$\Delta P = \frac{4T}{r} \Rightarrow \frac{\Delta P_1}{\Delta P_2} = 4$$

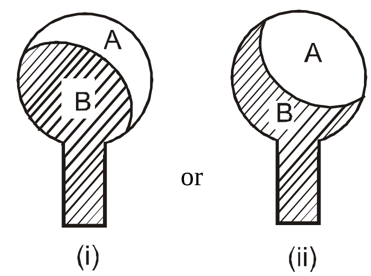
$$\therefore\ \frac{r_2}{r_1} = 4 \text{ and } \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{64}$$

Q.15 (3)

$$\Delta P = \frac{2T}{R} = \frac{2\times 70\times 10^{-3}}{1\times 10^{-3}} = 140\ N/m^2$$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)
After the portion A is punctured' the thread has 2 options as shown in the figures.



Clearly, due to surface tension , the soap film wants to minimize the surface area which is happening in option (ii).
Hence the thread will become concave towards A.

Q.2 (3)
We know that surface energy
$U_S = T \times \text{Area}$.
Here. as 2 films are formed because of ring. so
$U_S = T \times 2 \times (A)$

$$= 5\ \frac{N}{m} \times 2 \times 0.02\ m^2. = 0.2\ J$$

Q.3 (4)
Insects use the surface tension force to keep floating.

Q.4 (3)

$n \times \dfrac{4}{3}\ \pi r^3 = \dfrac{4}{3}\ \pi R^3$ ......(i) { $\because$ volumes are equal

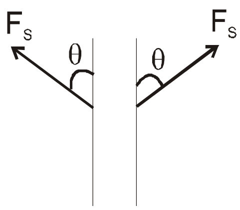
and $\Delta A = -[4\pi R^2 - n.4\pi r^2]$
where $W = (\Delta A) \times T$.
$= -4\pi[n^{2/3}r^2 - n.r^2] \times T = 4\pi r^2T.\ n^{2/3}\ [n^{1/3}–1]$.
Now $R^2 = n^{2/3}\ .\ r^2$; so $W = 4\pi R^2T[n^{1/3} - 1]$.

Q.5 (4)

In the satellite, g_{eff} becomes zero but the surface tension still prevails. Hence the water will experience only surface Tension force which will push it fully outward.

Q.6 (1)

Since the contact angle in both cases remains the same.



$F_S \cos\theta = Mg \Rightarrow T \times 2\pi R \cos\theta = Mg$ ....(i)

after doubling the radius

$T \times 2\pi(2R)\cos\theta = M'g$ ....(ii)

$= M' = 2M.$

Q.7 (2)

Water will rise to a height more than h when downward force (mg_{eff}) becomes lesser than mg.

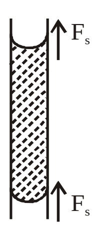
so in a lift accelerating downwards, g_{eff} is $(g - a_0)$. Hence capillary rise is more.

On the poles g_{eff} is even more than g. Hence the capillary will even drop.

Q.8 (1)

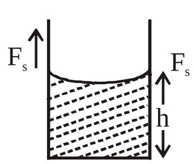
When the capillary rise is 'h' that means the force of surface tension (F) is supporting the height 'h' of liquid level.

Now if the whole capillary is taken out the liquid tries to come out due to gravity from the bottom point.



But force of surface tension 'F' now becomes 2F in the upward direction. Hence 2F can support a maximum of '2h' height even if ℓ is very high. So 'h' will be 2h if $\ell > h$ & will be $h + \ell$ only if ℓ is lesser than h.

Q.9 (2)



By balancing forces $T \times (2\ell) \times (\cos\theta) = d \times \ell\, h\, g$

we get $h = \dfrac{2T\cos\theta}{xdg}$.

Q.10 (1)

Energy released = $(\Delta A) \times \sigma$ {σ = surface tension}

Let us say n no. of small drops coalesced.

$$\Rightarrow n.\frac{4}{3}\pi a^3 = \frac{4}{3}\pi b^3$$

$$\Rightarrow b = a.n^{1/3} \qquad \Rightarrow n = \left(\frac{b}{a}\right)^3$$

$\Delta A = 4\pi b^2 - n.4\pi a^2$ {this is –ve, hence energy is released}

$= 4\pi a^2(n^{2/3} - n)$

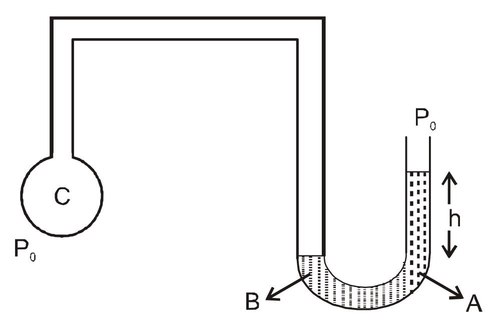
$$\Rightarrow U = 4\pi a^2 T(n - n^{2/3}) .= 4\pi a^2 T\left[\left(\frac{b}{a}\right)^3 - \left(\frac{b}{a}\right)^2\right]$$

This U converts to K.E.

$$\text{Hence } \frac{1}{2}\rho.\frac{4}{3}\pi b^3\ V^2 = 4\pi a^2 T\ \frac{b^2}{a^2}\left(\frac{b-a}{a}\right).$$

$$\Rightarrow V = \sqrt{\frac{6T}{\rho}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

Q.11 (4)



P_A has to be equal to P_B. $P_A = P_0 + \rho gh$ .....(i)

Now $P_C - P_0 = \dfrac{4\sigma}{r}$

$\because$ soap bubble has 2 films

and

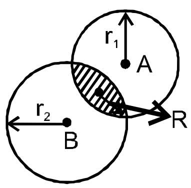
$P_C = P_B \because$ same air is filled

$\Rightarrow P_0 + \frac{4\sigma}{r} = P_0 + \rho gh$ ....(ii)

get $\sigma = \frac{\rho ghr}{4}$

Q.12 (3)
When charge is given to a soap bubble (whether positive or negative), these charges experience repulsive forces due to the other charges. Hence they tend to move out. Hence the size of bubble increases.

Q.13 (4)



Equating pressures on the shaded portion :

$$\frac{4\sigma}{r_1} - \frac{4\sigma}{r_2} = \frac{4\sigma}{R}$$

get $R = \frac{r_2 r_1}{r_2 - r_1}$

Q.14 (2)

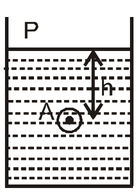
By equating volume : $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

get r = R/2.

Now pressure difference in A = $\frac{4\sigma}{R}$

and that in B = $\frac{4\sigma}{R/2}$ = 2 × pressure difference in A.

Q.15 (2)

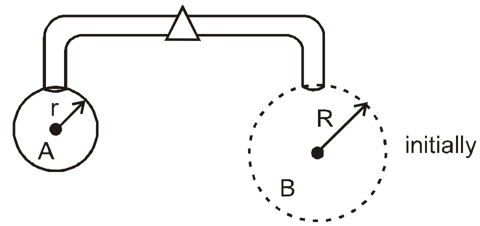


$P_{\text{inside bubble}} - P_A = \frac{2T}{r}$

and $P_A = P_{atm} + \rho gh$.

$\Rightarrow P_{\text{inside bubble}} = P + \rho gh + \frac{2T}{r}$

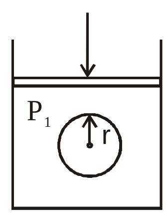
Q.16 (3)

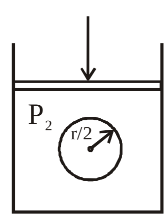


$P_A = P_0 + \frac{4\sigma}{r}$; $P_B = P_0 + \frac{4\sigma}{R}$ {P_0 = atmospheric pressure}.
Clearly $P_A > P_B$; so air will flow from A to B.
As r decreases; pressure will become more and hence more flow of air from A to B.
Ultimately bubble A collapses and B becomes bigger in size.

Q.17 (3)

Q.18 (1)

Before


After


Lets say, initially, the pressure due to air inside the bubble is P_{air}.

$\Rightarrow P_{air} - P_1 = \frac{4T}{r}$ ....(i)

Finally, the radius becomes half ; so volume becomes $\frac{1}{8}$ th and hence pressure becomes $8P_{air}$.

So, $8P_{air} - P_2 = \frac{4T}{r/2}$

.........(ii)

Solving (i) and (ii)

get $P_2 = 8P_1 + \dfrac{24r}{r}$.

Q.19 (4)

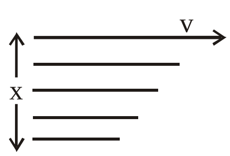
When the excess pressure at the hole becomes equal to the pressure of water height ;then only water will start coming out of the holes : [atm pressure on both sides is same].

$\Rightarrow \rho hg = \dfrac{2\sigma}{r}$

$\Rightarrow h = \dfrac{2\sigma}{\rho rg}$

$$= \frac{2\times70\times10^{-3}\times\dfrac{N}{m}}{1000\dfrac{kg}{m^3}\times\left(\dfrac{0.1}{2}\right)\times10^{-3}\times10} = 0.28\ \mathbf{m.}$$

Q.20 (3)



$800 = \eta A.\dfrac{1.5}{x}$

$2400 = \eta A\ \dfrac{v}{x}$

v = 4.5 cm /sec.

Q.21 (4)

$F_R = K.\pi r^2 . v^2 = \rho\dfrac{4}{3}\pi r^3 g$

$v \alpha \sqrt{r}$

Q.22 (2)

$2\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$R = 2^{1/3}.\ r$

$v \propto r^2$

$v \propto 4^{1/3}$

Q.23 (4)

$V_T = \dfrac{2}{9}\dfrac{r^2 g}{\eta}(\rho - \sigma)$

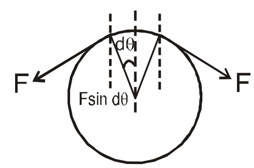
$= \dfrac{2}{9}\dfrac{(0.003)^2\times10}{1.260}\ (1260)$

$v_T = 0.02$ m /sec.

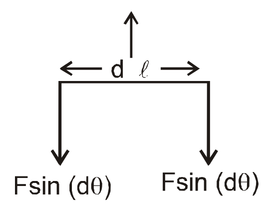
$\therefore$ Time $= \dfrac{0.1}{0.02} = 5$ sec.

JEE-ADVANCED OBJECTIVE QUESTIONS

Q.1 (D)



The small portion of film is approximately a straight part. Balancing forces on it:



F denotes tension. T denotes surface tension.

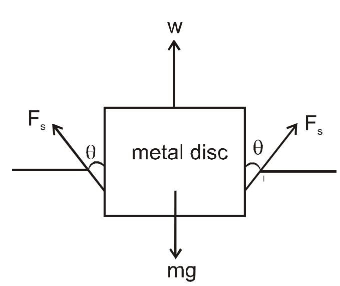
$T \times 2\,(d\ell)$ is the surface tension force because 2 layers are formed.

So $2F\sin(d\theta) = T\times[2\times R\,(2\,d\theta)]$

we get ; ($\sin(d\theta) \approx d\theta$. for small $d\theta$)

so $F = T\times 2R$.

Q.2 (C)



The FBD of disc is shown in the figure. The net upward surface tension force

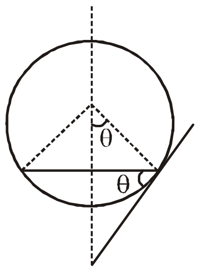
$= F_S\cos\theta = (T\times 2\pi r)\cos\theta$.

so $F_S\cos\theta + W = mg = W_{disc}$

$= F_S \cos\theta \ = \ (T \times 2\,\pi r)\cos\theta$.

$F_S \cos\theta \ + W = mg \ = W_{disc}$

Q.3 (B)

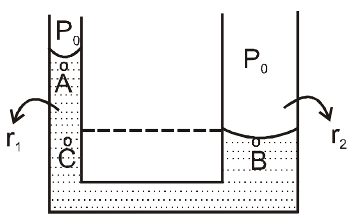


$$\cos\theta = \frac{R-h}{R}$$

$$\theta = \cos^{-1}\left(\frac{R-h}{R}\right)$$

Q.4 (A)

In the shown diagram.



$P_C = P_B$

$$P_0 - \frac{2T}{r_1} \ + \ \rho gh = P_0 - \frac{2T}{r_2}$$

Here, we may not know in advance which tube will rise above the other, but lets say the liquid level is higher in thinner tube.

$$\text{so } 2T\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -\rho gh.$$

$$\Rightarrow T = \frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$$

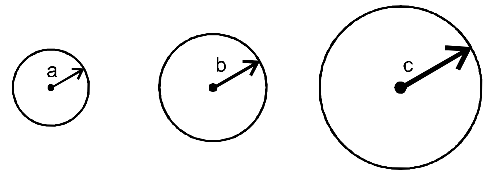
as $r_2 > r_1$; so we assumed correctly

Q.5 (B)

$$\frac{2T}{R} = h\rho g$$

Radius of Meniscus

Q.6 (A)



Let (a) and (b) coalesce to form (c).

By mole conservation :

$P_a \,.\, a^3 + P_b \,.\, b^3 = P_c \,.\, c^3$ . ... (i)

Also $P_a = P_0 + \dfrac{4\gamma}{a}$ ....(ii)

$P_b = P_0 + \dfrac{4\gamma}{b}$ ....(iii)

$P_c = P_0 + \dfrac{4\gamma}{c}$ ....(iv)

Putting there values :

$$\left(P_0 + \frac{4\gamma}{a}\right)a^3 + \left(P_0 + \frac{4\gamma}{b}\right)b^3 = \left(P_0 + \frac{4\gamma}{c}\right)c^3$$

$$\Rightarrow \ P_0\left[a^3 + b^3 - c^3\right] + 4\gamma\left[a^2 + b^2 - c^2\right] = 0$$

also $c^3 - (b^3 + a^3) = \dfrac{3v}{4\pi}$ and $c^2 - (a^2 + b^2) = \dfrac{s}{4\pi}$.

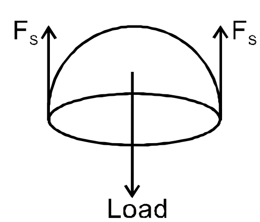
Putting there values :

$$P_0\left(\frac{-3v}{4\pi}\right) + 4T\left(\frac{-S}{4\pi}\right) = 0 \ \Rightarrow 3P_0V + 4ST = 0$$

Q.7 (B)

Clearly the surface tension force on

A soap - bubble with a radius ‘r’ is placed on another bubble with a radius R (figure). Angles between

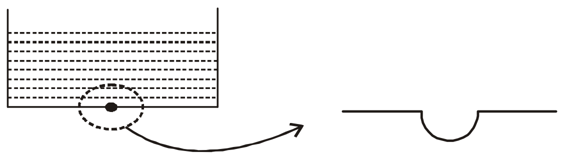


Hemisphere = $F_S = (2T).(2\pi r)$ {2 layers are formed}.

$$\Rightarrow \ F_S = 2 \times 500\ \frac{N}{m} \times 2 \times 3.14 \times 5m.$$

$$\approx 30{,}000\ N \approx 3000\ kg.wt.$$

Q.8 (C)



$$h\rho g = \frac{2T \times 2}{0.1}$$

$$h \times 1000 \times 10 = \frac{2 \times 75 \times 10^{-3} \times 2}{0.1 \times 10^{-3}}$$

$$h = \frac{300}{1000} - 0.3\text{ m} = 30\text{ cm.}$$

Q.9 (C)

$$P_{exi} = \frac{4T}{R}$$

$$4\pi R^2 d = \frac{4}{3}\pi r^3$$

$$(3R^2d)^{1/3} = r$$

$$P_{ef} = \frac{2T}{r} \quad \therefore \quad \text{Ratio} = \frac{R}{2r} = \left(\frac{R}{24d}\right)^{1/3}$$

Q.10 (A)

$$\eta\, a^2 \frac{v}{t} = mg \sin 37^o$$

$$\eta\, a^2 \frac{v}{t} = \rho a^3 g.\frac{3}{5}$$

$$\eta = \frac{3\rho agt}{5v}$$

Q.11 (A)

$$a_{avg} = \frac{v_T - 0}{T}$$

$$a_{air} = \frac{mg - f_B}{m} = g - \frac{F_B}{m} = g - \frac{v\,\rho_L\, g}{v\,\rho_S\, g} = g - \frac{\rho_L}{\rho_S} g$$

$$a_{air} = \frac{g(\rho_s - \rho_L)}{\rho_s} = \frac{2v_T}{T}$$

$$a_{air} = \frac{g(\rho_s - \rho_L)}{\rho_s} = 2.\frac{2}{9}\frac{r^2}{\eta}.\frac{9}{T}(\rho_s - \rho_r)$$

$$T = \frac{4}{9}\frac{\rho r^2}{\eta}$$

Q.12 (A)

$mg = m'g + 6\,\pi\,\eta\, r\, v$

$$\frac{m - m'}{r} \times v$$

Q.13 (C)

$$a = g - \frac{v\rho_L g}{m} - 6\pi\,\eta\, r\, v$$

straight line

Q.14 (D)

Q.15 (B)

$$F = \eta A \frac{dv}{dx}$$

$$= 1 \times 100 \times 10^{-4} \times \frac{7 \times 10^{-2}}{1 \times 10^{-3}}$$

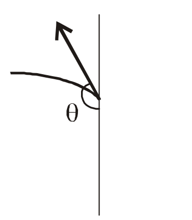
$$F = 0.7\text{ N}$$

JEE-ADVANCED

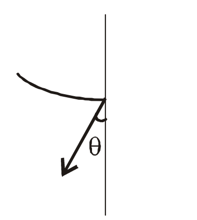
MCQ

Q.1 (A,B,C)

Force of cohesion keeps the molecules of a material bounded together and does not let them stick to the solid as force of adhesion is lesser.



Cohesion more than adhesion



adhesion more than cohesion

Hence

Q.2 (A, B, D)

Nature of liquid and material tube determine whether force of cohesion is more or force of adhesion is more. The inner radius also determines the rise of capillary as

$$h = \frac{2T\cos\theta}{r\rho g} \quad \text{depends on radius r.}$$

If the length is not sufficient rise will be depends length also.

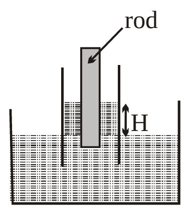
Q.3 (A, D)

When ever two drops coalesce to make a bigger drop. surface area is reduced, hence energy is released.

NUMERICAL VALUE BASED

Q.1 [6]

$$2\pi (r_1 + r_2) T = \rho(\pi(r_2^2 - r_1^2))gH$$



$$\Rightarrow \; H = \frac{2T}{(r_2 - r_1)\rho g}$$

$$\Rightarrow \quad H = 6\,\text{cm}$$

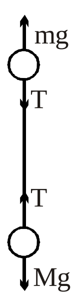
Q.2 [574]

Q.3 [1300]

Q.4 [4]

Q.5 [8]

$Mg - T - 6\pi\eta r_1 v = 0$



$mg + T - 6\pi\eta r_2 v = 0$

$$\frac{\frac{4}{3}\pi\left(r_1^3 + r_2^2\right)\times \rho g}{6\pi\eta(r_1 + r_2)} = v$$

$$v = \frac{2}{9}(r_1^2 - r_1 r_2 + r_2^2)\,\frac{\rho g}{\eta}$$

$$T = Mg - 6^2\pi\eta r_1 \times \frac{2}{9}(r_1^2 - r_1 r_2 + r_2^2)\,\frac{\rho g}{\eta}$$

$$= \frac{4}{3}\pi r_1^3 \times \rho g - \frac{4\pi}{3}\rho g\,[r_1^3 - r_1^2 r_2 + r_2^2 r_1]$$

$$= \frac{4}{3}\pi g r\,[r_1^2 r_2 - r_2^2 r_1]$$

$$= \frac{4}{3}\pi r g r_1^2\left[r_2 - \frac{r_2^2}{r_1}\right]$$

$$\frac{dT}{dr_2} = 0 \Rightarrow \frac{4}{3}\pi g r_1^2\left[1 - \frac{2r_2}{r_1}\right] = 0$$

$r_1 = 2r_2$

$$\frac{M}{m} = 8$$

Q.6 [50]

$$6\pi\eta r v = B = \frac{4}{3}\pi r^3 P_L\,g$$

$$\eta = \frac{2}{9} r^2\,\frac{P_L g}{v}$$

$$= \frac{2}{9} \times \frac{(0.9)^2 \times 1.75 \times 1000}{0.7} = 50 \text{ poise}$$

KVPY
PREVIOUS YEAR'S

Q.1 (A)

$$mg' - kv^2 = \frac{mdv}{dt}$$

Q.2 (C)

Check dimensionally

Q.3 (D)

Let X is thickness of soap film for equilibrium.

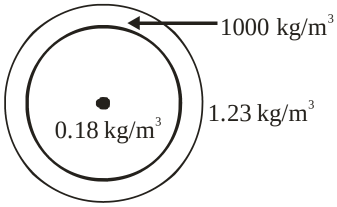
Gravity force = buoyancy force

$$\frac{4}{3}\pi(10^{-2})^3 \times 0.18 + 4\pi(10^{-2})^2$$

$$\Rightarrow \frac{4}{3}\pi(10^{-2})^3\,(1.23)$$

$$\Rightarrow \frac{4}{3}\pi(10^{-2})\,(x)\,(1000)$$

$$\Rightarrow \frac{4}{3}\pi(10^{-6}) \qquad (1.08)$$



$\Rightarrow \; (10^5)\,x = 0.36$

$\Rightarrow \; x = 0.36 \times 10^{-5}$

$\Rightarrow \; x = 3.6 \times 10^{-6}\,\text{m}$

Q.4 **(C)**

Due to soap bubble surface tension is reduced therefore in that area. Black paper powder will sink.

Q.5 (D)

$$\frac{1}{2}\rho v \times \pi R^2 = 4\pi RT \Rightarrow v = \sqrt{\frac{8T}{\rho R}}$$

(Here in question v is asked)

Q.6 (A)

$F_a = K\rho_a v^2 R^2 = K\rho_a v^2 M^{2/3}$ & $W = Mg$

When velocity becomes constant

$W = F_a$

$\Rightarrow Mg = K\rho_a v^2 M^{2/3}$

$\Rightarrow v^2 \propto M^{1/3} \Rightarrow v \propto M^{1/6}$

$$\Rightarrow \frac{v_1}{v_2} = \left(\frac{250}{125}\right)^{1/6} \Rightarrow \frac{v_1}{v_2} = (2)^{1/6}$$

Q.7 (B)

The steel ball will get terminal velocity when the net force on the ball is zero. So, in distance–time graph, slope become constant.

From graph :

$$V = \frac{0.4 - 0.3}{1.9 - 1.6} \approx 0.33 \text{ m/s}$$

JEE-MAIN PREVIOUS YEAR'S

Q.1 (1)

$$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\left(n^{1/3}\right) r = R$$

Δu loss

= T (change in surface area)

$= T(n4\pi r^2 - 4\pi R^2)$

= T4ƒà (nr^2 – R^2).

$$\Delta U = 4\pi T\left[\left(\frac{R}{r}\right)^3 r^2 - R^2\right]$$

$$\Delta U = 4\pi T\,\frac{\left[\frac{R^3}{r} - R^2\right]}{J}$$

$$\frac{\Delta U}{V} = \frac{4\pi T\left[\frac{R^3}{r} - R^2\right]}{J \times \frac{4}{3}\pi R^3} = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Q.2 (4)

The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta} = \frac{\rho (Q/A) D}{\eta} = \frac{\rho Q D}{A\eta}$$

$$\left[\begin{array}{ll} \rho \rightarrow \text{density of fluid} \quad ; & \eta \rightarrow \text{coefficient of} \\ v \rightarrow \text{velocity of flow} & \text{viscosity} \\ D \rightarrow \text{Diameter of pipe} & \end{array}\right]$$

From NCERT

If $Re < 1000$	$\rightarrow$ flow is steady
$1000 < Re < 2000$	$\rightarrow$ flow becomes unsteady
$Re > 2000$	$\rightarrow$ flow is turbulent

$$R_{e\,\text{initial}} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 382.16$

$$R_{e\,\text{final}} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 1019.09$

Q.3 (1)

Excess pressure at common surface is given by

$$P_{ex} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T}{r}$$

$$\therefore \; \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$$

$$r = \frac{ab}{b - a}$$

Q.4 (3)

Q.5 (1)

Q.6 (3)

Q.7 (2)

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Q.1 (D)

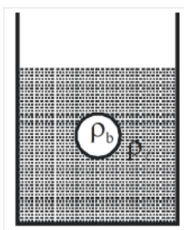
Q.2 (AD)

Consider a body of density ρ_b kept in density ρ_ℓ whose viscosity is h and terminal velocity V. Then

$$\vec{F}_{viscous} + \vec{F}_{mg} + \vec{F}_{buoyancy} = 0$$

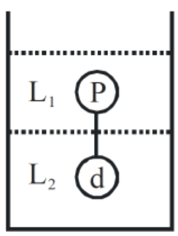
$$\vec{F}_{viscous} + \rho_b \frac{4}{3}\pi R^3(-\hat{j}) + \rho\ell \frac{4}{3}\pi R^3(\hat{j}) = 0$$

$$\therefore \vec{F}_{viscous} = (\rho_b - \rho_\ell)\frac{4}{3}\pi R^3(\hat{j}) \Rightarrow 6\pi\eta RV = (\rho_6 - \rho\ell)\frac{4}{3}\pi R^3$$



$\therefore$ If $\rho_b > \rho_1$ then $\vec{F}_{viscous} \uparrow V \propto \frac{1}{\eta}$ & if $\rho_b < \rho_\ell$

$\vec{F}_{vis\,cos} \downarrow$



as per given diagram we can say

$\sigma_2 > \sigma_1$; $\rho_1 < \sigma_1$ & $\rho_2 > \sigma_2$

$\Rightarrow \rho_2 > \sigma_2 < \sigma_1 > \rho_1$

$\therefore$ If we put P in L_2 where $|\vec{V}_P| \propto \frac{1}{\eta_2}$ when $\rho_1 < \sigma_2$

$\therefore \vec{F}_{vis\,cous} \downarrow \quad \therefore \vec{V}_P \uparrow$

$\therefore$ If we put Q in L_1 where $|\vec{V}_Q| \propto \frac{1}{\eta_1}$ when $\rho_2 < \sigma_1$

$\therefore \vec{F}_{vis\,cous} \uparrow \quad \therefore \vec{V}_P \downarrow$

$$\Rightarrow \frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2} \ \& \ \vec{V}_P \cdot \vec{V}_Q < 0$$

Q.3 [3]

$$V_T = \frac{2r^2 g(\sigma - \rho)}{9\eta}$$

$$\therefore \frac{V_P}{V_Q} = \frac{1^2(8-0.8)}{3\times\left(\frac{1}{2}\right)^2(8-1.6)\times\frac{1}{2}} = 3$$

Q.4 [6]

$R = K^{1/3}r$

$\Delta U = S.K.4\pi r2 - S.4\pi R^2$

$$\Delta U = 4\pi S\left[K.\frac{R^2}{K^{2/3}} - R^2\right]$$

$= 0.1 \times 10^{-4}[K^{1/3} - 1] = 10^{-3}$

$K^{1/3} - 1 = 10^2$

$K^{1/3} = 101 = (10^\alpha)^{1/3}$ $\alpha = 6$

Q.5 (A,C)

$\frac{2\sigma}{R} = \rho gh$ [R = Radius of meniscus]

$h = \frac{2\sigma}{R\rho g}$ $R = \frac{r}{\cos\theta}$

[r = radius of capillary, θ = contact angle]

$h = \frac{2\sigma\cos\theta}{r\rho g}$

(A) for given material, θ = constant

$h \propto \frac{1}{r}$

(B) h depend on σ

(C) if lift is going up with constant acceleration, $g_{off} = (g + a)$

$h = \frac{2\sigma\cos\theta}{r\rho(g+a)}$ It means h decreases.

(D) h is proportional to cosθ Not θ

Q.6 (A, C, D)

Viscous force is given by $F = -\eta A\frac{dv}{dy}$ since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A\frac{Dv}{\Delta y}$$

$$\therefore F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta \ \& \ F \propto u_0;$$

$$F \propto \frac{1}{h}, F \propto A$$

Since plate is moving with constant velocity, same force must be acting on the floor.

Q.7 (A,C,D)

$$h = \frac{2T\cos\theta}{\rho g R}; h_1 = \frac{2\times 0.075\times \cos 0^\circ}{1000\times 10\times 0.2\times 10^{-3}}$$

$\Rightarrow h_1$ = 75 mm (in T1) [If we assume entire tube of T1]

$\Rightarrow h_2 = \dfrac{2\times 0.075\times \cos 60^\circ}{1000\times 10\times 0.2\times 10^{-3}}$ = 37.5 mm (in T2) [If we assume entire tube of T2]

Option (1) : Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases, so option (1) is correct

Option (2) : If joint is 5 cm is above water surface, then lets say water crosses the joint by height h, then:

$$\Rightarrow P_0 - \frac{2T}{r} + \rho gh + \rho g \times 5 \times 10^{-2}$$
$$= P_0$$

$$\Rightarrow \cos\theta = \frac{R}{r}, r = \frac{R}{\cos\theta}$$

$$\Rightarrow \rho g\,(h + 5 \times 10^{-2}) = \frac{2T\cos\theta}{R}$$

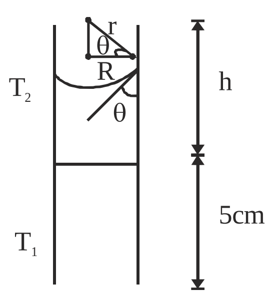
$$\Rightarrow h = \frac{2\times 0.075\times \cos 60}{0.2\times 10^{-3}\times 1000\times 10} - 5\times 10^{-2}$$

$\Rightarrow$ h = –ve, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

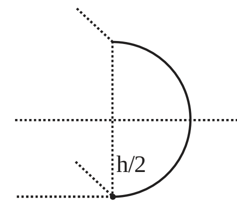
So option (2) is wrong.

Option (3) : If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T1, so option (3) is right.

Option (4) : If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (4) is right.



Q.8 [3.74]



Pressure at the bottom of disc =pressure due to surface tension

$$\rho gh = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$R_1 >>> R_2$

So $\dfrac{1}{R_1} <<< \dfrac{1}{R_2}$ and R_2=h/2

$$\therefore \rho gh = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = T\left(0 + \frac{1}{h/2}\right)$$

$$h^2 = \frac{2T}{\rho g}$$

$$h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{2\times 0.07}{10^3\times 10}} = \sqrt{\frac{14\times 100}{10^4\times 100}}$$

$$h = \sqrt{14}\text{mm} = 3.741$$

Q.9 (A,B,C)

$n_1 >> (n_1 - n_2) = \Delta n$

$$p_1 = \frac{n_1 RT}{N_A} \qquad p_2 = \frac{n_2 RT}{N_A}$$

$F = (n_1 - n_2) k_B TS = \Delta n k_B TS$ (A)

$$V = \frac{\Delta n\, k_B TS}{\beta}$$

Force balance $\Rightarrow$ Pressure ×Area = Total number of molecules × βv

$$\Delta n\, k_B TS = \ell n_1 S\beta v$$

$$\Rightarrow n_1\beta v\ell = \Delta n k_B T \qquad \text{(B)}$$

$$\text{Total number of molecules/sec} = \frac{(n_1 v\, dt)S}{dt}$$

$$= n_1 vS = \frac{\Delta n\, k_B TvS}{\beta v\ell}$$

$$= \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right)S \qquad \text{(C)}$$

As Δn will decrease with time therefore rate of molecules getting transfer decreases with time.